

**Ministry of Higher Education and  
Scientific Research**

**University of Technology  
Chemical Engineering Department**

# **Transport Phenomena**

*BY*

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## References

Colulsson ,J.M and Richardson J.F. "Chemical Engineering , volume 1", 3ed edition ,Robert Maxwell.M.C.

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## **syllabus**

### **1. Boundary Layer and analogies:**

Boundary Layer in Laminar and turbulent Flow, Boundary Layer in Laminar sub Layer, Transition layer, Velocity distribution on Surfaces and Pipes, Momentum, heat and mass transfer molecular diffusion, Eddy transfer, Reynolds analogy, Modified Reynolds analogy, Chilton and Colburn analogy.

(20 hrs)

### **2. Evaporation:**

Introduction, Types of Evaporations, Evaporation Equipment, Heat transfer in Evaporation Process, single, double and Multi effect Evaporators, Design of evaporators, Comparison of Forward, backward and Parallel effect evaporators, boiling Point rise.

(12 hrs)

### **3. Drying:**

Introduction and general Principle in drying, Rate of drying, the mechanism of moisture movement, Calculation of rate of drying, moisture transport in Solids at Constant and falling rate Period, Capillary movement, Material and Energy Balances in Continuous dryers, Types of Dryers

(8 hrs)

### **4. Humidification, dehumidification and Cooling towers:**

Humidification, dehumidification, Temperature humidity Chart for air – water system . Enthalpy – humidity –temperature chart. Addition of Vapor or liquid Stream to a gas Stream. Mechanism of dehumidification Evaluation of heat and Mass transfer Coefficient, Cooling tower, height of Packing in Cooling towers, Minimum gas Condition, Change of Condition of liquid and gas in Cooling towers.

(14 hrs)

### **5. Extraction:**

Definition, Extraction process, Equilateral Triangular coordinates (Ternary Diagram), system of three liquid \_ one pair partially soluble, system of three liquid \_ two, pairs partially soluble, choice of solvent, Equipment in extraction cross \_ current extraction, multi stage Cross Current extracting cross current for insoluble Liquid , Continuous Counter current extraction , Continuous Counter Current in Soluble , Liquid , Minimum Solvent , Counter Current extraction with reflux .

(22 hrs)

### **6. Filtration:**

Type of Filters, Filtration theory, Plate and frame filter press, leaf filter, filtration at Constant  $\Delta P$ , Filtration at Constant rate, washing Time.

(10 hrs)

### **7. Crystallization:**

Introduction Equilibrium Solubility in crystallization, Classification of Crystallizes, Nucleation theory, Rate of Crystal Growth, Material and Energy balance in Crystallization.

(4 hrs)

# Transport Phenomena

## CHAPTER 1

### *Drying of Solids*

#### **1.1 Introduction.**

Drying refers to the removal of water, or another solute "liquid", to reduce the content of residual liquid to an acceptably low value . The drying of materials is often the final operation in a manufacturing process, carried out immediately prior to packaging or dispatch. Drying is the operation often follows evaporation, filtration, or crystallization. In some cases, drying is an essential part of the manufacturing process, as for instance in paper making or in the seasoning of timber.

**Drying is carried out for one or more of the following reasons:**

- (a) To reduce the cost of transport.
- (b) To make a material more suitable for handling as, for example, with soap powders, dyestuffs and fertilizers.
- (c) To provide definite properties, such as, for example, maintaining the free-flowing nature of salt.
- (d) To remove moisture which may otherwise lead to corrosion. One example is the drying of gaseous fuels or benzene prior to chlorination.

#### **1.2. General Principles.**

The moisture content of a material is usually expressed in terms of its water "liquid" content as a percentage of the mass of the dry material, though moisture content is sometimes expressed on a wet basis. If a material is exposed to air at a given temperature and humidity, the material will either lose water "if the air have lower humidity than that corresponding to the moisture content of the solid " or gain water "if air has more humid than the solid in equilibrium with it ,the solid absorbs moisture from the air" until an equilibrium condition is established.

Moisture may be present in the following forms:-

##### ❖ **Bound Moisture.**

This is the moisture "water" "contained by a substance that it exerts a vapor pressure less than that of free water at the same temperature. Bound water may be exist in several conditions such as water retained in small

capillaries, adsorbed on surfaces, or as a solution in cell walls and in organic substance in the physical and chemical combination.

❖ **Unbound Moisture.**

This is the moisture "water" contained by a substance which exerts a vapor pressure as high as that of free water at the same temperature and is largely held in the voids of solid.

❖ **Equilibrium Moisture Content  $X^*$ .**

Is the portion of the water in the wet solid which can not be removed by the inlet air.

❖ **Free Moisture  $X$ .**

This is water which is in excess of the equilibrium moisture content.

Where  $X = X_t - X^*$ ,  $X_t$  is the total moisture content

The water removed by vaporization is generally carried away by air or hot gases, and the ability of these gases to pick up the water is determined by their temperature and humidity. In designing dryers using air, the properties of the air–water system are essential.

A non-porous insoluble solid, such as sand or china clay, has an equilibrium moisture content approaching zero for all humidities and temperatures, although many organic materials, such as wood, textiles, and leather, show wide variations of equilibrium moisture content.

For the *air–water system*, the following definitions are of importance:

- *Humidity  $\mathcal{H}$* , mass of vapor "water" per unit mass of dry air.

$$\mathcal{H} = \frac{M_A P_A}{M_B (P - P_A)}$$

$P_A$  – Partial pressure of water vapor.

$P$  – Total pressure.

$M_A$  – Molecular weight of water vapor;  $M_B$  –Molecular weight of air

- *Humidity of saturated air  $\mathcal{H}_0$*  . This is the humidity of air when it is saturated with water vapor. The air then is in equilibrium with water at the given temperature and pressure.

$$\mathcal{H}_0 = \frac{M_A P_A^0}{M_B (P - P_A^0)}$$

$P_A^0$  - water vapor pressure

- *Percentage humidity  $\mathcal{H}_p$*

$$= \frac{\text{Humidity of air}}{\text{Humidity of saturated air}} \times 100 = \frac{\mathcal{H}}{\mathcal{H}_0} \times 100$$

- Percentage relative humidity  $RH\%$

$$= \frac{\text{Partial pressure of water vapour in air}}{\text{Vapour pressure of water at the same temperature}} \times 100$$

Figure (1) show the moisture content as a function of Percentage relative humidity

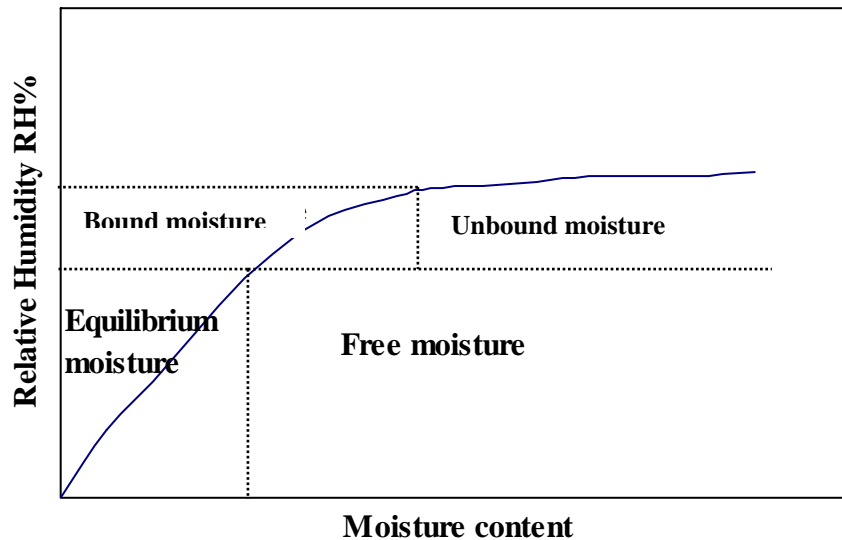


Figure (1) moisture content vs. Percentage relative humidity.

Consider a sample of wool has 26% moisture at 100% relative humidity, consequently this sample if contain less than 26 % water , contains only bound water; any moisture that a sample contain above 26% is unbound water.

### 1.3 Rate of Drying.

The time required for drying of a moist solid to final moisture content can be determined from a knowledge of the rate of drying under a given set of conditions The drying rate of a solid is a function of temperature , humidity , flow rate and transport properties ( in terms of Reynolds number and Schmidt number) of the drying gas .

In drying, it is necessary to remove free moisture from the surface and also moisture from the interior of the material. If the change in moisture content for a material is determined as a function of time (see figure (2)), a smooth curve is obtained from which the rate of drying at any given moisture content may be evaluated. The form of the drying rate curve varies with the structure and type of material, and two typical curves are shown in Figure 2. In curve 1, there are two well-defined zones: AB, where the rate of drying is constant and BC, where there is a steady fall in the rate of drying as the moisture content is reduced. The moisture content at the end of the constant rate period is represented by point B,

and this is known as the *critical moisture content*. Curve 2 shows three stages, DE, EF and FC. The stage DE represents a constant rate period, and EF and FC are falling rate periods. In this case, the Section EF is a straight line, however, and only the portion FC is curved. Section EF is known as the first falling rate period and the final stage, shown as FC, as the second falling rate period. The drying of soap gives rise to a curve of type 1, and sand to a curve of type 2.

At zero time, the initial free moisture content is shown at point D" or D' depend on the initial solid enters the dryer, If the solid is at colder temperature than its ultimate temperature, and the evaporation rate will increase. Eventually at point D the surface temperature rises to its equilibrium value. Alternatively, if the solid is quite hot to start with, the rate may start at point D'. This unsteady state adjustment often ignored in the analysis of times of drying.

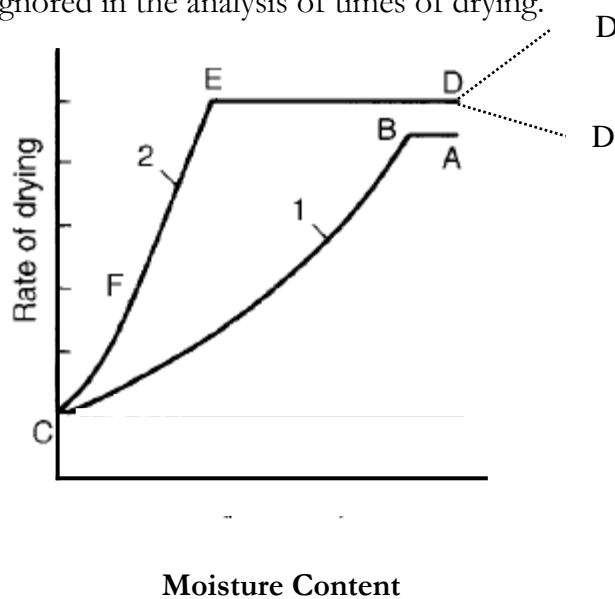


Figure (2) Rate of drying of a granular material.

### **Constant rate period**

During the constant rate period, it is assumed that drying takes place from a saturated surface of the material by diffusion of the water vapour through a stationary air film into the air stream.

### **First falling-rate period**

The points B and E in Figure (2) represent conditions where the surface is no longer capable of supplying sufficient free moisture to saturate the air in contact with it. Under these conditions, the rate of drying depends very much on the mechanism by which the moisture from inside the material is transferred to the surface. In general, the curves in Figure (2) will apply, although for a type 1 solid, a simplified expression for the rate of drying in this period may be obtained.

### **Second falling-rate period**

At the conclusion of the first falling rate period it may be assumed that the surface is dry and that the plane of separation has moved into the solid. In this case,

evaporation takes place from within the solid and the vapour reaches the surface by molecular diffusion through the material. The forces controlling the vapor diffusion determine the final rate of drying, and these are largely independent of the conditions outside the material.

## 1.4 Calculation method of Drying Rate and Time.

### 1.4.1 Drying Rate.

#### \* Method used Experimental Drying Curve.

Data obtained from batch drying experiments are usually obtained as W (total weight of the wet solid "dry + moisture") at different times t (h) in the drying period.

$$X_t(\text{dry basis}) = \frac{W - L_s}{L_s}$$

Where  $L_s$  weight of dry solid.

For a given constant drying condition, the equilibrium moisture content  $X^*$  is determined then the free moisture is calculated for each value of  $X_t$ .

$$X = X_t - X^*$$

Plot  $X$  vs.  $t$ , to obtain the rate of drying ( $R$ ) from this plot, the slope of the tangents drawn can be measured which give the value  $dX/dt$  at a given value of  $t$ . The rate  $R$  for each point is :-

$$R = -\frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 1$$

Where  $R$  rate of drying  $\text{Kg}_{\text{H}_2\text{O}}/\text{h.m}^2$ .

$A$  Exposed surface area for drying  $\text{m}^2$ .

The drying rate curve is then obtained by plotting  $R$  vs. the moisture content.

Another method to obtain the value of drying curve is to calculate the weight loss  $\Delta X$  for a  $\Delta t$  time. For example; if  $X_1=0.35$  at  $t_1=1.68\text{h}$  and  $X_2=0.325$  at  $t_2=2.04\text{h}$ , then:-

$$\frac{\Delta X}{\Delta t} = \frac{0.35 - 0.325}{2.04 - 1.68}$$

and if  $\frac{L_s}{A} = 2.15$

Therefore;  $R = \frac{L_s}{A} \frac{\Delta X}{\Delta t} = 1.493$

This rate is the average over the period 1.68h to 2.04h and should be plotted at the average concentration.

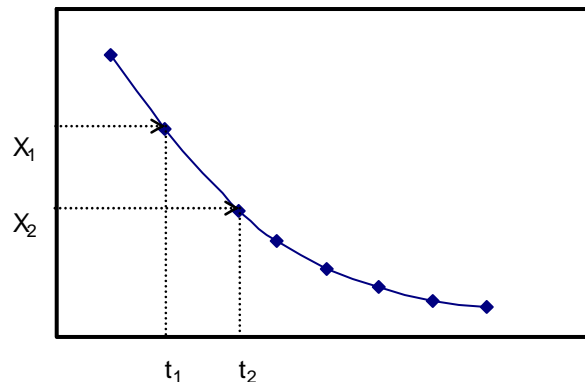


## 1.4.2 Drying Time.

### 1. Constant Rate Drying Period.

#### ❖ Method used Drying Curve.

To estimate the time of drying for a given batch of material ,based on actual experimental data. The time required is determined directly from the drying curve of free moisture content vs. time.



Where  $t = t_2 - t_1$  and  $X_1, X_2 > X_c$  (critical moisture content)

#### ❖ Method used Drying Rate Equation

$$R = -\frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 1$$

Rearranged (1) and integrated over the time interval to dry from  $X_1$  at  $t_1=0$  to  $X_2$  at  $t_2=t$

$$t = \int_{t_1=0}^{t_2=t} dt = \frac{L_s}{A} \int_{X_2}^{X_1} \frac{dX}{R} \dots\dots\dots 2$$

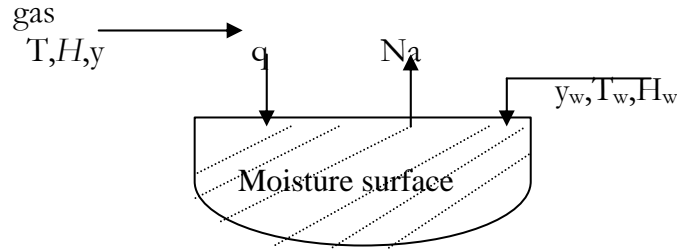
Drying takes place within the constant rate period ,then  $R = \text{constant} = R_c$  .

$$t = \frac{L_s}{AR_c} (X_1 - X_2) \dots\dots\dots 3$$

In the constant rate period of drying , the surface of grains of solid in contact with drying air flow remain completely wetted. Drying of material occurs by mass transfer vapor from the saturated surface of the material through the air film to the bulk gas phase .The rate of moisture movement within the solid is sufficient to keep the surface saturated. The rate removal of water vapor (drying) is controlled by the rate of heat transfer to the evaporating surface which furnishes the latent heat of evaporation for the liquid. At steady state the rate of mass transfer balances the rate of heat transfer.

Assuming only heat transfer to the solid surface by convection from the hot gas to the surface of the solid and mass transfer from the surface to the hot gas.

- ❖ No heat transfer by conduction from metal pans or surface , neglect heat transfer by radiation.



The rate of heat transfer by convection ( $q$ ) from the gas at  $T$  to the surface of the solid at  $T_w$  is :-  $q = hA(T - T_w)$ .....4

The flux of water vapor from the surface is :-

$$Na = k_y (y_w - y) \text{.....5a}$$

$$\text{or} \quad Na = k_y \frac{M_B}{M_A} (H_w - H) \text{.....5b}$$

where  $Na$  mass flux kmol/m<sup>2</sup>.sec

$$\text{Where } y = \frac{H/M_A}{1/M_B + H/M_A},$$

the term  $(H/M_A)$  neglected due  $H$  very small.....so:-

$$y = \frac{H M_B}{M_A}, \quad y_w = \frac{H_w M_B}{M_A}$$

$$\begin{aligned} \text{The amount of heat needed } q &= m \cdot \lambda_w \\ &= Na \cdot M_A \cdot A \cdot \lambda_w \text{.....6} \end{aligned}$$

Where  $\lambda_w$  latent heat of vaporization at  $T_w$ .

Equating equation(4) and(6) and sub into(5b)

$$R_c = \frac{q}{A \lambda_w} = \frac{h(T - T_w)}{\lambda_w} = k_y M_B (H_w - H) \text{.....7}$$

Therefore equation (3) become :-

$$\begin{aligned} t &= \frac{L_s \lambda_w}{A h (T - T_w)} (X_1 - X_2) \\ t &= \frac{L_s}{A k_y M_B (H_w - H)} (X_1 - X_2) \end{aligned}$$

To predict  $R_c$ , the heat transfer coefficient  $h$  must be known.

- if air flowing parallel to the drying surface:-

$$h = 0.0204 G^{0.8} \quad SI \text{ unit}$$

$$h = 0.0128 G^{0.8} \quad English \text{ unit}$$

at  $T_{air}$  45-150 °C,  $G=2450-2950 \text{ kg/m}^2\cdot\text{h}$ ,  $U_g=0.61-7.6 \text{ m/s}$

- if air flowing perpendicular to the drying surface :-

$$h = 1.17 G^{0.37} \quad SI \text{ unit}$$

$$h = 0.37 G^{0.37} \quad English \text{ unit}$$

$G=3900-19500 \text{ kg/m}^2\cdot\text{h}$ ,  $U_g=0.9-4.6 \text{ m/s}$

Where gas mass velocity  $G=\rho_{air}\cdot v_{air}$

$$\rho_{air} = \left( \frac{1+H}{v_H} \right)$$

$$\begin{aligned} v_H (\text{humid volume}) &= \frac{22.4 \cdot T}{273} \left( \frac{1}{M_A} + \frac{H}{M_B} \right) \\ &= (2.83 \cdot 10^{-3} + 4.56 \cdot 10^{-3} H) T, \quad T \text{ in K} \end{aligned}$$

## 2. Falling Rate Drying Period.

### • Graphical Integration Method.

In falling rate period, the rate of drying  $R$  is not constant but decreases when drying proceeds past the critical free moisture content  $X_c$ . The time for drying for any region between  $X_1$  and  $X_2$  given by :-

$$t = \frac{L_s}{A} \int_{X_2}^{X_1} \frac{dX}{R} \dots\dots\dots 8$$

In falling rate period,  $R$  varies for any shape of falling rate drying curve, equation(8) can be integrated graphically by plotting  $1/R$  vs.  $X$  and determine the area under the curve.

### • Calculation Method for special cases in Falling Rate Period.

#### 1. Rate is a linear function of $X$ .

Both  $X_1$  and  $X_2 < X_c$  and the rate  $R$  is linear in  $X$  over this region.

$$R = a X + b \dots\dots\dots 9$$

Where  $a$  is a slope of the line and  $b$  is a constant .Differentiating equation(9) gives:-

$$dR = a dX \text{ .....sub into equation(8)}$$

$$t_f = \frac{L_s}{a A} \int_{R_2}^{R_1} \frac{dR}{R}$$

$$t_f = \frac{L_s}{a A} \ln \frac{R_1}{R_2} \text{ .....10}$$

$$\text{Since } R_1 = a X_1 + b$$

$$R_2 = a X_2 + b$$

$$\text{Since } a = \frac{R_1 - R_2}{X_1 - X_2}$$

$$\text{So that } t_f = \frac{L_s (X_1 - X_2)}{A (R_1 - R_2)} \ln \frac{R_1}{R_2} \text{ .....11}$$

## 2. Rate is a Linear Function Through Origin.

In some cases a straight line from the critical moisture content passing through the origin adequately represents the whole falling rate period .Then the rate of drying is directly proportional to the free moisture content.

$$R = a X \text{ .....12}$$

Differentiating equation(12)

$$dR = a dX \text{ ..... } dX = \frac{dR}{a} \text{ .....Sub in equation(8)}$$

$$t_f = \frac{L_s}{a A} \int_{R_2}^{R_1} \frac{dR}{R} = \frac{L_s}{a A} \ln \frac{R_1}{R_2}$$

The slope **a** of the line is  $R_c/X_c$  and for  $X_1=X_c$  at  $R_1=R_c$

$$\text{Then; } t_f = \frac{L_s X_c}{A R_c} \ln \frac{R_c}{R_2} \text{ .....13}$$

Noting also that  $R_c/R_2 = X_c/X_2$

Therefore equation (13) become:-

$$t_f = \frac{L_s X_c}{A R_c} \ln \frac{X_c}{X_2} \text{ .....14}$$

## 1.5 Mechanism of Moisture Movement During Falling Rate Period.

In the falling rate period , the surface of the solid being dried is no longer completely wetted , and the rate of drying steadily falls with time. The rate of moisture movement in the falling rate period is governed by the rate of internal movement of the liquid by "liquid diffusion" or "capillary movement".

### 1.5.1 Liquid Diffusion of Moisture in Drying.

When liquid diffusion of moisture controls the rate of drying in the falling rate period. The equation for diffusion used Fick's second law for unsteady state diffusion using the concentrations as X kg free moisture/kg dry solid instead of concentrations kg mole moisture /m<sup>3</sup>,

$$\frac{dX}{dt} = D_L \frac{d^2 X}{dx^2} \dots\dots\dots 15$$

Where D<sub>L</sub> liquid diffusion coefficient m<sup>2</sup>/h.

x distance in the solid m.

This type of diffusion is often characteristic of relating slow drying in non-granular materials such as soap ,gelatin and others. During diffusion type drying ,the resistance to mass transfer of water vapor from the surface is usually very small and the diffusion in the solid controls the rate of drying .Then the moisture content at the surface is at the equilibrium value X\* . This means that the free moisture content X at the surface is zero.

Assume that initial moisture distribution is uniform at t=0 ,equation(15) integrated to give :-

$$\frac{X_t - X^*}{X_1 - X^*} = \frac{X}{X_1} = \frac{8}{\Pi^2} \left[ e^{-D_L t \left( \frac{\Pi}{2x_1} \right)^2} + \frac{1}{9} e^{-9D_L t \left( \frac{\Pi}{2x_1} \right)^2} + \frac{1}{25} e^{-25D_L t \left( \frac{\Pi}{2x_1} \right)^2} + \dots \right] \dots\dots 16$$

Where X average free moisture content at time t.

X<sub>1</sub> initial free moisture content at time =0.

X\* equilibrium free moisture content.

x<sub>1</sub> =0.5 thickness of the slab when drying occurs from the top and the bottom parallel faces, and x<sub>1</sub>=total thickness of slab if drying only from the top face.

From equation(16) assume that D<sub>L</sub> is constant ,but D<sub>L</sub> is rarely constant ,it varies with moisture content ,temperature, and humidity. For long drying times ,only the first term in the equation 16 is significant:-

$$\frac{X}{X_t} = \frac{8}{\Pi^2} e^{-D_L t \left(\frac{\Pi}{2x_1}\right)^2} \dots\dots\dots 16b$$

Solving for the time of drying:-

$$t = \frac{4x_1^2}{\Pi^2 D_L} \ln \frac{8X_1}{\Pi^2 X} \dots\dots\dots 17$$

In this equation if the diffusion mechanism starts at  $X=X_c$ . Differentiating equation (17) with respect to time and rearranging ,

$$\frac{dX}{dt} = \frac{-\Pi^2 D_L X}{4 x_1^2}$$

Multiplying both sides by  $L_s/A$ :-

$$R = -\frac{L_s}{A} \frac{dX}{dt} = \frac{-\Pi^2 L_s D_L X}{4 x_1^2 A} \dots\dots\dots 18$$

Equation (17 and 18) state that when internal diffusion controls for long times , the rate of drying is directly proportional to the free moisture  $X$  and the liquid diffusivity and that the rate of drying is inversely proportional to the thickness squared.

### 1.5.2 Capillary movement of Moisture in Drying.

Water can flow from region of high concentrations to those of low concentrations as a result of capillary action rather than by diffusion if the pore size of granular materials are suitable . The capillary theory assumes that a packed bed of non-porous spheres contains a void space between the spheres called pores. As water is evaporated ,capillary forces are set up by the interfacial tension between the water and solid. These forces provide the driving force for moving the water through the pores to the drying surface.

If the moisture movement follows the capillary theory ,the rate of drying  $R$  will vary linearly with  $X$  :-

$$R = -\frac{L_s}{A} \frac{dX}{dt} \dots\dots\dots 1$$

For the rate  $R$  varying linearly with  $X$  ;then  $R = R_c(X / X_c)$  ; therefore ;-

$$t_f = \frac{L_s X_c}{A R_c} \ln \frac{X_c}{X_2} \dots\dots\dots 14$$

Where  $L_s = x_1.A.\rho_s$  kg dry solid

$\rho_s$  Solid density kg/m<sup>3</sup>

$x_1$  Thickness m

$A$  Exposed surface area m<sup>2</sup>

Equation 14 become :-

$$t_f = \frac{x_1 \rho_s X_c}{R_c} \ln \frac{X_c}{X_2} \dots\dots\dots 19$$

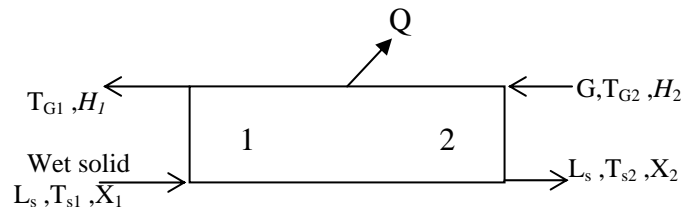
Where  $R_c$  can be calculated from equation (7) :-

$$R_c = \frac{q}{A \lambda_w} = \frac{h(T - T_w)}{\lambda_w} = k_y M_B (H_w - H) \dots\dots\dots 7$$

The time of drying is directly proportional to the thickness , while the rate of drying inversely proportional to the thickness when capillary controls the falling rate period.

## 1.6 Material and Heat Balance for Continuous Dryers.

Figure below show a flow diagram for a continuous type dryer where the drying gas flows counter currently to the solids flow.



The solid enters at a rate of  $L_s$  kg<sub>dry solid</sub>/h, having a free moisture content  $X_1$  and a temperature  $T_{s1}$ . It leaves at  $X_2$  and  $T_{s2}$ . The gas enters at a rate  $G$  kg<sub>dry air</sub>/h, having a humidity  $H_2$  kg<sub>water vapor</sub>/kg<sub>dry air</sub> and a temperature of  $T_{G2}$ . The gas leaves at  $T_{G1}$  and  $H_1$ .

For **material balance** on the moisture :-

$$GH_2 + L_s X_1 = GH_1 + L_s X_2 \dots\dots\dots 20$$

For **heat balance** :- the enthalpy of wet solid is composed of the enthalpy of the dry solid plus that of free liquid as free moisture. The heat of wetting is usually neglected .

The enthalpy of **gas  $H_G$**  in kJ/kg<sub>dry air</sub> is:-

$$H_G = S (T_G - T_o) + \lambda_o H \dots\dots\dots 21$$

$$S = C_a + C_H H \text{ ,kJ/kg.K} \dots\dots\dots 22$$

Where

$\lambda_o$  latent heat of vaporization of water (liquid)at reference temperature=2501 kJ/kg.

$S$  humid heat kJ/kg.k.

$C_a$  and  $C_H$  specific heat of gas and vapor kJ/kg.K.

The enthalpy of **wet solid  $H_s$**  in kJ/kg<sub>dry solid</sub> is :-

$$H_s = C_{ps} (T_s - T_o) + X C_{pA} (T_s - T_o) \dots\dots\dots 23$$

Where

$C_{ps}$  Specific heat of dry solid kJ/kg dry solid.k.

$C_{pA}$  Specific heat of liquid moisture kJ/kg water.k.

The heat of wetting or adsorption is neglected. So the heat balance on the dryer is :-

$$GH_{G2} + L_s H_{s1} = G H_{G1} + L_s H_{s2} + Q$$

Where Q is the heat loss in the dryer kJ/kg. For adiabatic process  $Q=0$  , and if heat is added Q is negative

## 1.7 Drying Equipment.

Some dryers are continues ,and some operate batch wise ,some agitate the solids ,and some are essentially un-agitated. Operation under vacuum may be used to reduce the drying temperature. Some dryers can handle almost any kind of material, while others are severally limited in the type of feed they can be accept.

A major division may be made between :-

1. Dryers in which the solid is directly exposed to a hot gas.
2. Dryers in which heat is transferred to the solid from an external medium such as condensing steam ,usually through a metal surface with which the solid is not in contact.

Dryers which exposed the solids to a hot gas are called direct dryers; those in which heat is transferred from an external medium are known indirect dryers.

Solids handling in dryers, most industrial dryer handle particulate solids during part or all of the drying cycle ,in adiabatic dryers the solids are exposed to the gas in the following ways :-

1. Gas is blown through a bed of coarse granular solids which are supported on a screen. This is known as through – circulation drying. As in cross-circulation drying the gas velocity is kept low to avoid any entrainment of solid particles.
2. Solids are showered downward through a slowly moving gas stream, often with some undesired entrainment of fine particles in the gas.
3. Gas is blown across the surface of the bed or slab of solids, or across one or both faces of a continuous sheet or film. This process is called cross-circulation drying.
4. Gas passes through the solids at a velocity sufficient to fluidize the bed.
5. The solids are all entrained in a high-velocity gas stream and are pneumatically conveyed from a mixing device to a mechanical separator.



In indirect dryers heating the only gas to be removed is the vaporized water or solvent ,although sometimes a small amount of "sweep gas" often air or nitrogen is passed through the unit.

1. Solids are spread over a stationary or slowly moving horizontal surface and cooked until dry. The surface may b heated electrically or by a heat transfer fluid such as steam or hot water.
2. solids are moved over a heated surface ,usually cylindrical by an agitator or a screw or paddle conveyor.
3. Solids slides by a gravity over an inclined heated surface or are carried upward with the surface for a time and then slides to a new location  
"Rotary dryer"

# Transport Phenomena

## CHAPTER 2

### *Humidification, Dehumidification, and Cooling Tower.*

#### **2.1 Introduction.**

Humidification and dehumidification involves simultaneous transfer of material between a pure liquid phase and a fixed gas which is insoluble in the liquid .

**Humidification :-** is a process to increase the amount of vapor present in the gas stream, by passing the gas over a liquid which then evaporates into gas stream. This transfer into the main stream takes place by diffusion, and at the interface simultaneous transfer of heat and mass. The drying of wet solid is an example of humidification process.

**Dehumidification :-** is a process to reduce the vapor present in gas stream. In this operation ,partial condensation must be affected and the condensed vapor removed. Dehumidification has use in air conditioning.

#### **2.2 Terminology and Definitions.**

Temperature ,humidity and enthalpy of gas (air) are the more important quantities in dealing with humidification. The rate of transfer of water vapor (generally to the air) depends upon the vapor pressure of the water and the moisture concentration in air. Two types of temperature the "dry bulb" and the "wet bulb" temperature are defined in connection with air-water contacting. Moisture concentration is expressed in term of "humidity".

*Humidity  $\mathcal{H}$* , mass of vapor "water" per unit mass of dry air.

$$\mathcal{H} = \frac{M_A P_A^0}{M_B (P - P_A)}$$

$P_A$  – Partial pressure of water vapor.

$P$  – Total pressure.

$M_A$  – Molecular weight of water vapor;  $M_B$  –Molecular weight of air

*Humidity of saturated air  $\mathcal{H}_0$*  . This is the humidity of air when it is saturated with water vapor. The air then is in equilibrium with water at the given temperature and pressure.

$$\mathcal{H}_0 = \frac{M_A P_A^0}{M_B (P - P_A^0)}$$

Percentage relative humidity  $RH\%$

$$= \frac{\text{Partial pressure of water vapour in air}}{\text{Vapour pressure of water at the same temperature}} \times 100$$

$$RH\% = \frac{P_A}{P_A^o} \times 100$$

100% humidity means saturated gas, and 0% humidity means vapor free gas

Percentage humidity  $H\%$

$$= \frac{\text{Humidity of air}}{\text{Humidity of saturated air}} \times 100 = \frac{H}{H_0} \times 100$$

$$\begin{aligned} &= \frac{P_A M_A / M_B (P - P_A)}{P_A^o M_A / M_B (P - P_A^o)} \times 100 \\ &= \frac{P_A / (P - P_A)}{P_A^o / (P - P_A^o)} \times 100 = RH \times \frac{(P - P_A^o)}{(P - P_A)} \times 100 \end{aligned}$$

*Humid volume  $v_H$*  . Is the volume of unit mass of dry air with accompanying water vapor at a given temperature and pressure.(assume ideal gas where volume of 1 kmole of a gas at 1 atm and 0 °C = 22.4 m<sup>3</sup>) .

$$v_H = \frac{22.4}{273} \left( \frac{1}{M_B} + \frac{H}{M_A} \right) \times T \text{ .....in m}^3/\text{kg} \text{.....}$$

Where  $M_B$  molecular weight of air 29

$M_A$  molecular weight of water vapor 18

$T$  temperature K

For vapor –free gas  $H=0$  ,  $v_H$  is the specific volume of the fixed gas. For saturated gas  $H=H_0$ ,  $v_H$  becomes the saturated volume.

*Humid heat  $S$*  . is the energy required to rise the temperature of unit mass of dry air and its associated water vapor through unit temperature difference at constant pressure .

$$S = C_a + C_w H \text{ ,kJ/kg.K}$$

$C_a$  and  $C_w$  is the specific heat of gas and vapor kJ/kg.K.

*Dry-Bulb temperature* . This is the temperature of air measured by a thermometer whose "bulb" is dry, i.e. not in touch with water or any other liquid. This is the true temperature of the air.

*Wet bulb temperature.* This is the temperature attained by a small amount of evaporating water in a manner such that the sensible heat transferred from the air to the liquid is equal to the latent heat required for the evaporation. The wet bulb temperature is measured by passing air over the bulb of a thermometer which is covered with a cloth – wick saturated water.

The rate of which this temperature is reached depends on the initial temperatures and the rate of flow of gas past the liquid surface with a small area of contact between the gas stream remain virtually unchanged.

The rate of transfer of heat from the gas to the liquid is :-

$$Q = h A (T_G - T_w) \dots\dots\dots 1$$

Where Q heat transfer rate.

h heat transfer coefficient.

$T_G$  dry bulb temperature.

$T_w$  wet bulb temperature.

The liquid evaporating into the gas is transferred by diffusion from the interface to the gas stream as a result of a concentration difference ( $C_o - C$ ).

Where  $C_o$  concentration of the vapor at the surface

C concentration in the gas stream.

$C_o$  and C mass per unit volume.

The rate of evaporation is :-  $m = h_D A (C_o - C) \dots\dots\dots 2a$

$$m = h_D A \frac{M_A}{RT} (P_{A_o} - P_A) \dots\dots\dots 2b$$

Where  $h_D$  is the mass transfer coefficient

$P_A$  and  $P_{A_o}$  partial pressure of air and vapor pressure of water vapor, and can be expressed in terms of the corresponding humidities H and  $H_w$ .

If  $P_A$  and  $P_{A_o}$  are small compared with P,  $(P - P_A)$  and  $(P - P_{A_o})$ , can be replaced by a main partial pressure of the gas  $P_B$ .

$$H = \frac{M_A P_A^0}{M_B (P - P_A)}$$

$$H_w = \frac{M_A P_A^0}{M_B (P - P_A^0)} \dots\dots\dots \text{humidity of gas saturated at the wet bulb temperature.}$$

$$H_w - H = \left[ \frac{P_A^0}{(P - P_A^0)} - \frac{P_A}{(P - P_A)} \right] \frac{M_A}{M_B}$$

$$H_w - H = \left[ \frac{P_A^0}{P_B} - \frac{P_A}{P_B} \right] \frac{M_A}{M_B} = (P_A^0 - P_A) \frac{M_A}{M_B}$$

$$P_A^0 - P_A = (H_w - H) P_B \frac{M_B}{M_A} \dots\dots\dots \text{sub into equation (2b)}$$

$$m = h_D A \frac{M_A}{RT} (H_w - H) P_B \frac{M_B}{M_A}$$

$$\text{Where } \rho_B = \frac{P_B M_B}{RT}$$

Therefore equations (2a) and (2b) become :-

$$m = h_D A \rho_B (H_w - H) \quad \dots\dots\dots 3$$

The heat transfer required to maintain this rate of evaporation is :-

$$Q = m \cdot \lambda \quad \dots\dots\dots 4a$$

$$\therefore Q = h_D A \rho_B (H_w - H) \lambda \quad \dots\dots\dots 4b$$

Where  $\lambda$  is the latent heat of vaporization of liquid.

Equating equations (1) and (4b) :-

$$Q = h A (T_G - T_w) = h_D A \rho_B (H_w - H) \lambda$$

$$H_w - H = \frac{h}{h_D \rho \lambda} (T_G - T_w) \quad \dots\dots\dots 5$$

Where  $-\frac{h}{h_D \rho} = S$  (humid heat) at moderate humidities

The above equation is known Lewis relation.

Heat and mass transport through the air film on the moist wick are illustrated in figure(1).

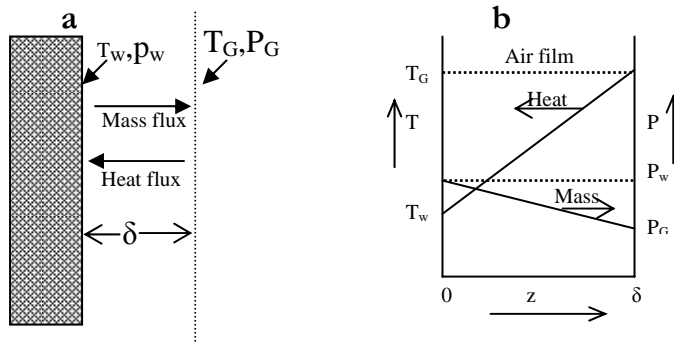


Figure (1):(a) schematic of the wick and air film;(b) temperature and partial pressure of water vapor profile in air film.

*Dew point.* If the vapor – gas mixture is gradually cooled at a constant pressure , the temperature at which it just becomes saturated is called its dew point. The partial pressure at water vapor in air is the same as the vapor pressure of water at this temperature. If the temperature is reduced even slightly, condensation occurs and droplets (dew) of water appear.

*Adiabatic saturation temperature*  $T_s$ . if the gas is passed over the liquid at such a rate that the time of contact is sufficient for equilibrium to be established, the gas will become saturated and both phases will be brought to the same temperature. In a thermally insulated system, the total sensible heat falls by an amount equal to the latent heat of the liquid evaporated. As a result of continued passage of the gas

,the temperature of the liquid gradually approaches an equilibrium value which is known adiabatic saturation temperature.

Make heat balance over the column,

$$\text{Enthalpy of the inlet air} = S (T_G - T_s) + H \lambda_s$$

$$\begin{aligned} \text{Enthalpy of exit air} &= S (T_s - T_s) + H_s \lambda_s \\ &= H_s \lambda_s \end{aligned}$$

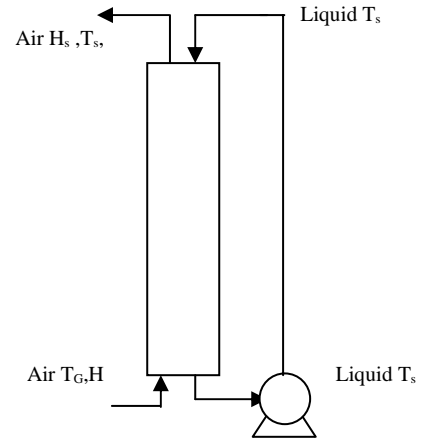
Where  $\lambda_s$  latent heat of vaporization of water at adiabatic saturation temperature.

Because water temperature dose not change , its enthalpy remain constant.

At steady state:

$$S(T_G - T_s) + H \lambda_s = H_s \lambda_s \dots\dots\dots 6$$

$$(T_G - T_s) = (H - H_s) \frac{\lambda_s}{S} \dots\dots\dots 7$$



## 2.3 Humidity Measurement.

The humidity of stream gas may be found by measuring either the dew point or wet bulb temperature or direct absorption method or by chemical method.

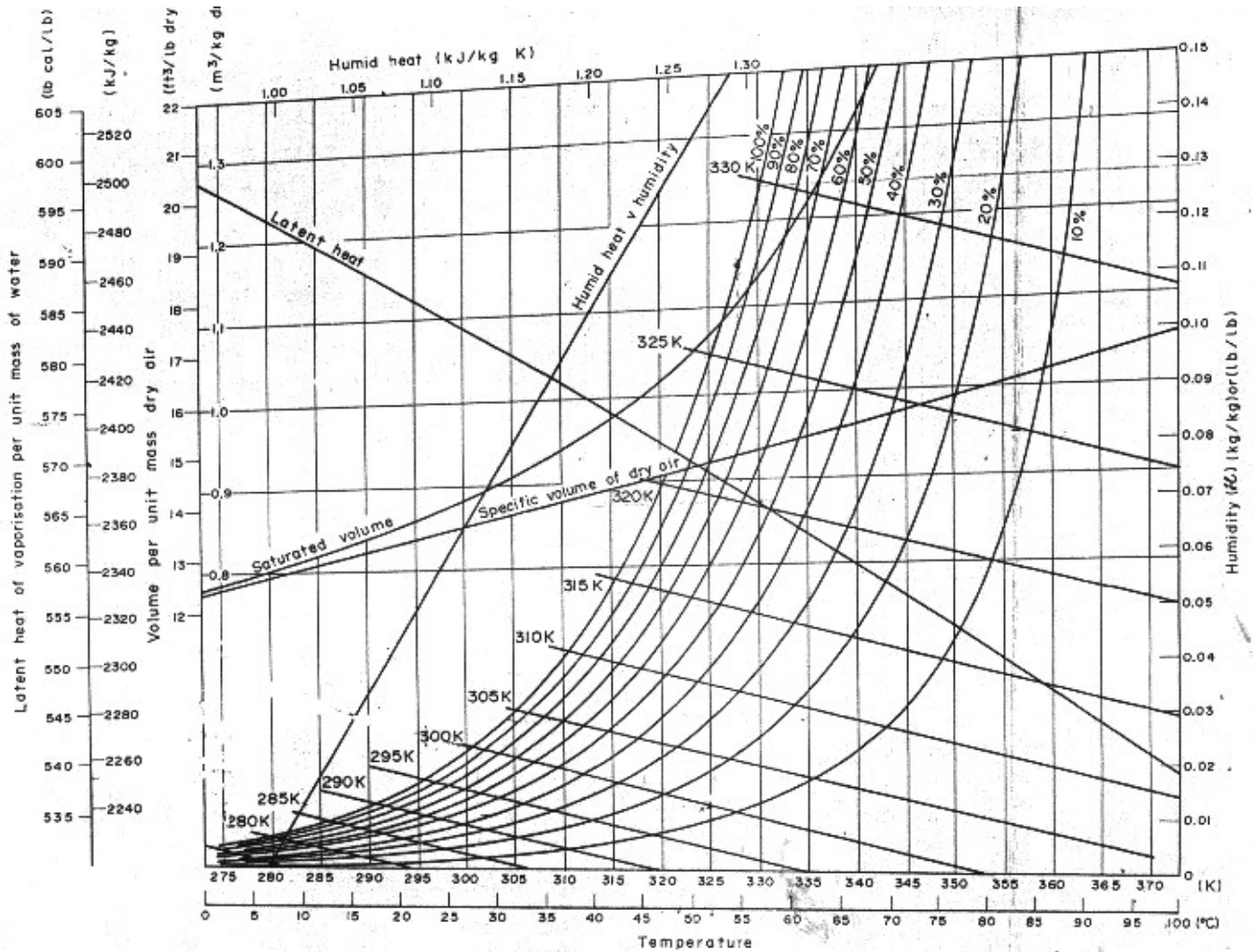
## 2.4 Humidity Data for Air – Water system.

Various type of humidity chart is available, based on either the temperature or enthalpy of the gas.

### ❖ Temperature –Humidity Chart (Psychrometric Chart).

From this chart(see figure 2) the following quantities can be estimated:-

1. Humidity (absolute and relative  $H$ ).
2. specific volume of dry gas.
3. saturated volume.
4. latent heat of vaporization.
5. humid heat.
6. dry and wet bulb temperature.
7. dew and saturation temperature.



Figure(2) Temperature –Humidity Chart (Psychrometric Chart) for air-water system.

If any two of the above quantities are known the others can be readily obtained. For a given wet bulb temperature equation (5) can be plotted on humidity chart

(H vs. T) as a straight line having a slope of  $-\frac{h}{h_D \rho \lambda}$  which known

**Psychrometric line.** Also equation (7) can be plotted which gave **adiabatic cooling (saturation) line** having the slope  $-\frac{S}{\lambda}$ .

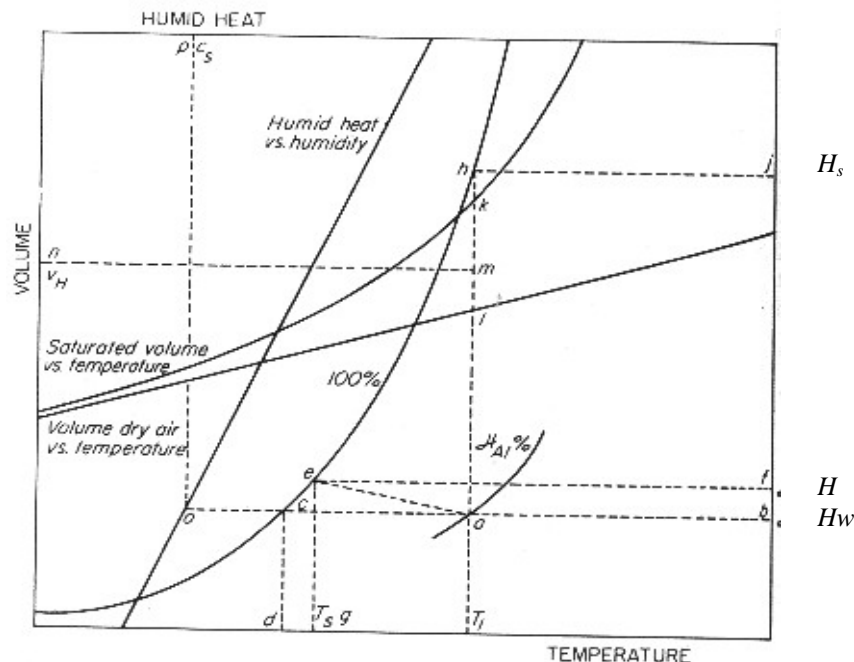
For air-water system  $T_s = T_w$  ;therefore,

$$-\frac{S}{\lambda} = -\frac{h}{h_D \rho \lambda} \dots\dots\dots 8$$

So the **adiabatic cooling (saturation) line** and **Psychrometric line** become essentially the same.

### Uses of Humidity Chart.

1. The air dry bulb temperature  $T_G$  and its humidity  $H$ , is known and denoted in the figure(3) by the point **a**. The humidity of air is denoted by point **b** from the humidity coordinate .
2. The dew point  $T_d$  , is found by the point **d** that can be reached by moving horizontally from the point **a** to the RH=100% line at **c** and then moving vertically down to the temperature axis **d**.
3. The adiabatic saturation line through **a** is **ae**( these are a series of almost parallel lines as shown in figures (2 and 3). The adiabatic saturation temperature is obtained by drawing the vertical line from **e** then down to the temperature axis **g**. For air -water system the wet bulb temperature is the same as adiabatic saturation temperature. The humidity of the adiabatically saturated air is given by the point **f**. Interpolation between the adiabatic lines may be necessary.
4. If the air originally is subsequent saturated at constant temperature, the humidity is found by following the temperature line through point **a** to point **h** on the RH=100% line and reading the humidity at point **j**.
5. The humid volume of saturated air at temperature  $T_G$  corresponds to the point **k**, and that of dry air at  $T_G$  is given by the point **i** .The point **m** gives the humid volume if the humidity is  $H$ . It reached by interpolation between **i** and **k**.
6. The humid heat of the air is found by locating point **O** ,the intersection of the humidity line through point **a** and the humid heat line ,and reading the humid heat at point **p** on the scale at the top.



Figure(3)Uses Temperature –Humidity Chart (Psychrometric Chart) for air-water system.



❖ **Enthalpy –Humidity Chart.**

Enthalpy of humid air means that 1 kg of dry air with accompanying water vapor has a heat content of **H** kJ/kg with respect to specified reference states of air and water. The reference states may be chosen arbitrarily .

**For saturated air:-**

$$\mathbf{H} = \mathbf{H}_a + \mathbf{H}_w .H \dots\dots\dots 9$$

Where

**H<sub>a</sub>** enthalpy of dry gas per unit mass, =  $C_a(T_G - T_o)$

**H<sub>w</sub>** enthalpy of vapor per unit mass, =  $C_w(T_G - T_o) + \lambda_o$

**C<sub>a</sub>** specific heat of the gas at constant pressure.

**C<sub>w</sub>** specific heat of the vapor at constant pressure.

**λ<sub>o</sub>** latent heat of vaporization of water (liquid) at reference temperature.

$$\mathbf{H} = C_a(T_G - T_o) + [C_w(T_G - T_o) + \lambda_o].H$$

$$= (C_a + C_w H).(T_G - T_o) + \lambda_o H$$

$$\mathbf{H} = S (T_G - T_o) + \lambda_o H \dots\dots\dots 10$$

If the gas contain more liquid or vapor than is required to saturate; the gas either will be supersaturated or the excess material will be present in the form of liquid or solid according to the temperature  $T_G$  is greater or less than the reference temperature  $T_o$ .

❖ If  $T_G > T_o$  and  $H > H_o$  of saturated gas, the enthalpy **H** per unit mass of dry gas is :-

$$\mathbf{H} = \underset{1}{C_a(T_G - T_o)} + \underset{2}{[C_w(T_G - T_o) + \lambda_o].H_o} + \underset{3}{C_L(H - H_o)(T_G - T_o)} \dots\dots\dots 11$$

The 1<sup>st</sup> term is the enthalpy of dry air.

The 2<sup>nd</sup> term is the enthalpy of saturation.

The 3<sup>rd</sup> term is the enthalpy of liquid in gas.

**C<sub>L</sub>** is the specific heat of liquid.

❖ If  $T_G < T_o$  the corresponding enthalpy is :-

$$\mathbf{H} = C_a(T_G - T_o) + [C_w(T_G - T_o) + \lambda_o].H_o + [C_s(T_G - T_o) + \lambda_f] (H - H_o) \dots\dots\dots 12$$

Where **C<sub>s</sub>** is the specific heat of solid.

**λ<sub>f</sub>** is the latent heat of freezing of liquid.

Two cases may be considered to illustrate the use of enthalpy – humidity charts. These are the mixing of two streams of humid gas and the addition of liquid or vapor to a gas.

**1. Mixing of Two Stream of Humid Gas.**

Consider the mixing of two gases of humidities  $H_1$  and  $H_2$ , temperatures  $T_{G1}$  and  $T_{G2}$  and the enthalpies  $\mathbf{H}_1$  and  $\mathbf{H}_2$  to give a mixed gas of temperature  $T_G$ , enthalpy  $\mathbf{H}$  and humidity  $H$ .

Taking a balance on the dry gas ,vapor and enthalpy :-

$$M_1 + M_2 = M \dots\dots\dots 13$$

$$M_1 H_1 + M_2 H_2 = M H \dots\dots\dots 14$$

$$M_1 \mathbf{H}_1 + M_2 \mathbf{H}_2 = M \mathbf{H} \dots\dots\dots 15$$

Elimination of M gives :-

$$M_1 (H - H_1) = M_2 (H_2 - H) \dots\dots\dots 16a$$

$$M_1 (\mathbf{H} - \mathbf{H}_1) = M_2 (\mathbf{H}_2 - \mathbf{H}) \dots\dots\dots 16b$$

Dividing these two equations :-

$$\frac{(H - H_1)}{(\mathbf{H} - \mathbf{H}_1)} = \frac{(H_2 - H)}{(\mathbf{H}_2 - \mathbf{H})} \dots\dots\dots 17$$

The condition of the resultant gas is therefore represented by a point on the straight line joining  $(H_1, \mathbf{H}_1)$  and  $(H_2, \mathbf{H}_2)$ . The humidity is given from equation 16a.

The gas formed by mixing two unsaturated gases may be either unsaturated ,saturated or supersaturated. The possibility of producing supersaturated gas arises because the 100% relative humidity line on the humidity – enthalpy chart is concave towards the humidity axis.

**2. Addition of Liquid or Vapor to a Gas.**

Let a mass  $M_3$  of liquid or vapor of enthalpy  $\mathbf{H}_3$  be added to a gas of humidity  $H_1$  and enthalpy  $\mathbf{H}_1$  and containing a mass of dry gas. Then :-

$$M_1 (H - H_1) = M_3 \dots\dots\dots 18$$

$$M_1 (\mathbf{H} - \mathbf{H}_1) = M_3 \mathbf{H}_3 \dots\dots\dots 19$$

$$\frac{(\mathbf{H} - \mathbf{H}_1)}{(H - H_1)} = \mathbf{H}_3 \dots\dots\dots 20$$

Where  $H$  and  $\mathbf{H}$  is the humidity and enthalpy of the gas produced in mixing. The composition and properties of the mixed stream are represented by a point on the straight line of slope  $\mathbf{H}_3$ , relative to the humidity axis which passes through the point  $(H_1, \mathbf{H}_1)$ .

In Figure (4), the edges of the origin, give a straight line of the slope indicated. In using the chart (Figure (4)) , a line of slope  $\mathbf{H}_3$  is drawn through the point  $(H_1, \mathbf{H}_1)$  . The point representing the final gas stream is then given from equation (18)

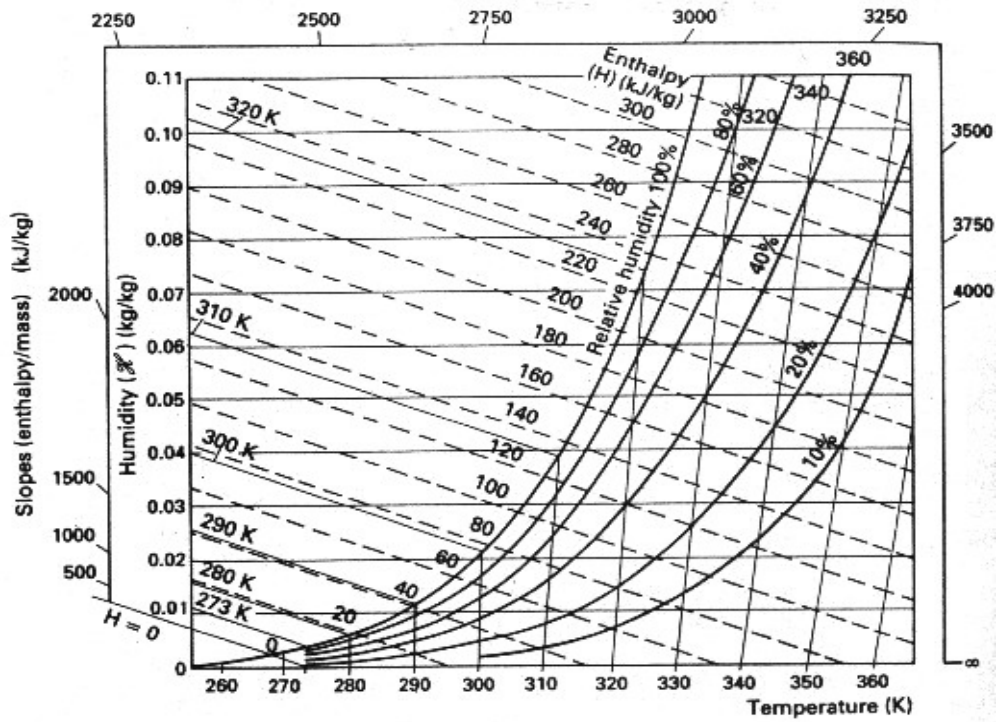


Figure (4) Enthalpy- Humidity chart for air-water system.

## 2.5 Theory of Humidification processes.

In an adiabatic humidifier, where the liquid remains at constant adiabatic saturation temperature, there is no temperature gradient through the liquid. In dehumidification and in liquid cooling, however, where the temperature of the liquid is changing, sensible heat flows into or from the liquid, and a temperature gradient is thereby setup. This introduces a liquid-phase resistance to the flow of heat. On the other hand, there can be no liquid-phase resistance to mass transfer in any case, since there can be no concentration gradient in a pure liquid.

### *Mechanism of Interaction of Gas and Liquid*

#### 1. Adiabatic humidification with liquid at constant temperature, is shown in figure(5)

The latent heat flow from liquid to gas just balances the sensible heat flow from gas to liquid, and there is no temperature gradient in the liquid. The gas temperature  $T_G$  must be higher than the interface temperature  $T_i$ . In order that sensible heat may flow to the interface; and  $H_i$  must be greater than  $H$  (humidity of the bulk gas) in order that the gas

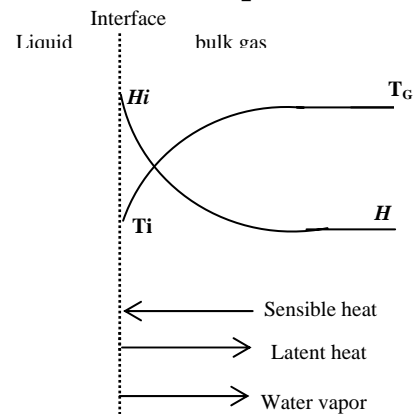


Figure (5) Adiabatic humidifier

#### 2. Condition in a dehumidifier as shown in figure(6).

Here Humidity  $H$  is greater than the interface humidity  $H_i$ , and therefore vapor must diffuse to the interface. Since the temperature interface  $T_i$  and humidity interface  $H_i$  represent saturated gas, bulk gas temperature  $T_G$  must be greater than  $T_i$ ; otherwise the bulk of the gas would be supersaturated with vapor.

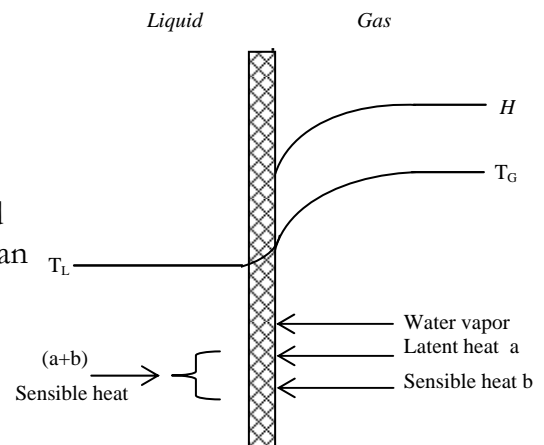


Figure (6)

## 2.6 Cooling Tower.

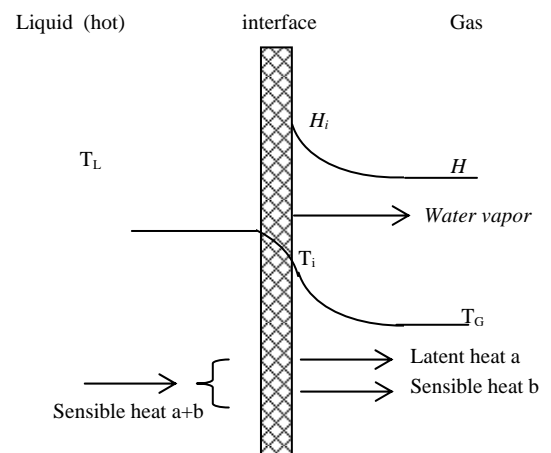
In a typical cooling tower, air and water are brought into countercurrent contact; the water flows down over a series of wooden slats or spray tower which give a large interfacial area and promote turbulence in the liquid. The flow of air upward through the tower can be induced by the buoyancy of the warm air in the tower

(natural draft) or by mechanical draft such as fan. Cooling takes place both by the transference of sensible heat and by evaporative cooling as a results of which sensible heat in the water provides the latent heat of vaporization. In the cooling (warm water) tower the temperature of liquid is falls and the temperature and humidity of the air is rise , and its action is thus similar to that of an air humidifier. The water can not be cooled below the wet bulb temperature corresponding to the condition of the air at inlet. The driving force for vapor pressure it would have at the wet bulb temperature. The water can be cooled only to the wet bulb temperature , in practice it is cooled to within 1K of the wet bulb temperature of inlet air. The enthalpy of the air stream dose not remain constant since the temperature of the liquid changes rapidly in the upper portion of the tower. Towards the bottom, however, the temperature of the liquid changes less rapidly because the temperature differences are smaller.

### ***Mechanism of Interaction of Gas and Liquid in Cooling Tower.***

**In a continuous cooling tower the conditions depends on whether the gas temperature is below or above the temperature at the interface.**

The upper part of the cooling tower is shown in figure (7). The flow of heat and of vapor (and hence the direction of temperature and humidity gradient) are exactly the reverse of the figure (6). The liquid is being cooled both by evaporation and by transfer of sensible heat, the humidity and temperature of the gas decrease in the direction of interface to gas, and the temperature drop through the liquid must sufficient to give a heat transfer rate high enough to account for both heat items.



Figure(7)

In the lower part of the cooling tower is shown in figure (8). The temperature of the gas is above that of the interface temperature. The liquid is being cooled; hence the interface must be cooler than the bulk of the liquid, and the temperature gradient through the liquid is toward the interface. On the other hand, there must be a flow of sensible heat from the bulk of the gas to the interface ( $T_G$  must greater than  $T_i$ ). The flow of vapor away from the interface carriers, as latent heat , all the sensible heat supplied to the interface from both sides.

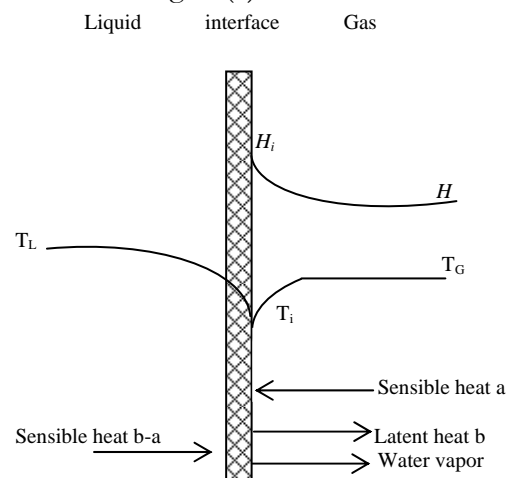


Figure (8)

## 2.7 Cooling Tower Calculation.

In an air contacting tower may have three major application:-

- i. Evaporation cooling of warm water from the cooling water circuit in a plant.
- ii. Cooling of hot gas by direct contact with cool water.
- iii. Dehumidification of a warm moist gas in contact with cool water.

### 2.7.1 Sizing of Cooling Tower.

For sizing the cooling tower, it primarily need to calculate:

- The tower cross-section required to take the given load of warm water.
- The height of packing required to achieve the desired cooling of the water.

Simultaneous heat and mass transfer are involved. Mass and enthalpy balance were made for the air and the water streams over a thin section of the tower and integrated these equations for the calculation of the required height of the packing. There is no concentration gradient in the liquid and therefore there is no resistance to mass transfer in the liquid phase.

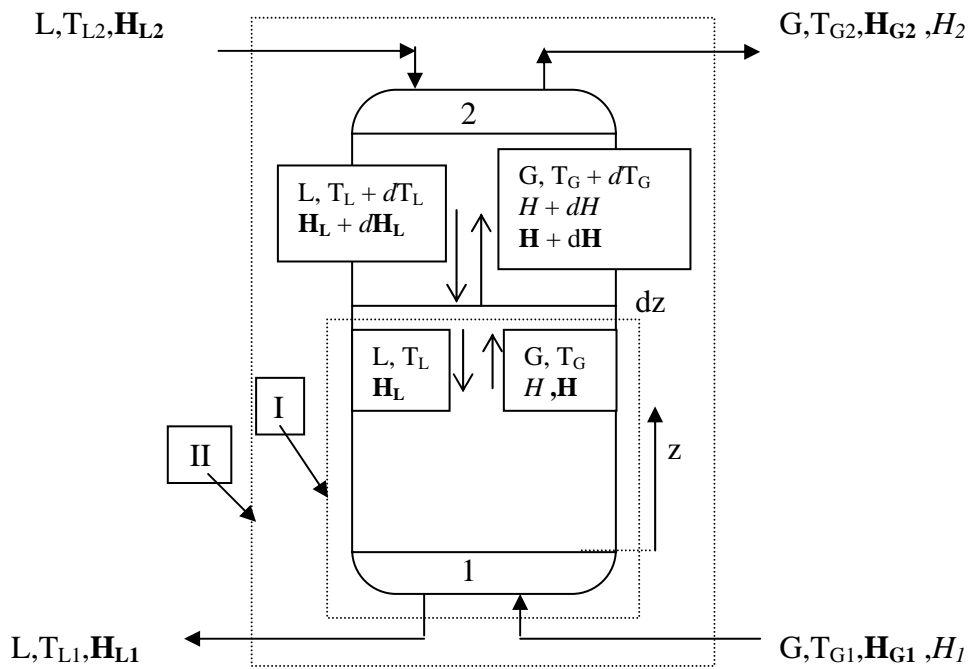


Figure (9) mass and enthalpy balance in a cooling tower.

- 1) Let  $L$  ( $\text{kg}/\text{m}^2\cdot\text{s}$ ) rate of water flow "constant",  $G$  ( $\text{kg dry air}/\text{m}^2\cdot\text{s}$ ) rate of air flow. The temperature of water decreases by  $dT_L$ , the enthalpy of the air increases by  $dH$  across a differential thickness  $dz$  of the bed.

$$\text{Change in enthalpy of water} = L \cdot dH_L = L \cdot C_L \cdot dT_L \quad \dots\dots\dots 1$$

$$\text{Change in enthalpy of air} = G \cdot dH_G \quad \dots\dots\dots 2$$

Where  $\mathbf{H_L} = C_L(T_L - T_o)$ .....3

$\mathbf{H_G} = S(T_G - T_o) + H\lambda$ .....4

$C_L$  specific heat of liquid

$S$  humid heat

From equation (3),  $d\mathbf{H_L} = C_L.dT_L$

**Enthalpy balance over the envelop I gives :-**

$$G.d\mathbf{H_G} = L.C_L.dT_L \text{.....5}$$

Integrating equation(5) over the entire tower ,give :-

$$L.C_L (T_{L2} - T_{L1}) = G (\mathbf{H_{G2}} - \mathbf{H_{G1}}) \text{ .....6}$$

Equation(6)is the **Operating Line** for air-water contact. Since L,G and  $C_L$  remain constant along the tower, it is a straight line on  $T_L$ - $\mathbf{H}$  plane having a slope of  $L.C_L/G$  .From equation (6) it appears that the operating line may obtained by joining the terminal points  $(T_{L1}, \mathbf{H_1})$  and  $(T_{L2}, \mathbf{H_2})$ . The equilibrium (saturation) curve for air-water system on the  $T_L$ - $\mathbf{H}$  plane is the plot of the enthalpy of saturated air vs. the liquid temperature at equilibrium.

2) The rate of transfer of water vapor from interface to the bulk gas(air) :-

$$h_D.p.a.dz (H_i - H) = G.dH \text{.....7a}$$

where  $h_D$  Individual mass transfer coefficient for the vapor.

$\rho$  Mean density of the air.

$H_i$  Humidity of air at the interface.

$a$  Interfacial area per unit volume of the column.

Rearranging equation (7a) :-

$$\frac{dH}{H_i - H} = \frac{h_D \rho a}{G} dz \text{ .....7b}$$

Multiply equation (7a) by  $\lambda$  "humid heat" :-

$$h_D.p. \lambda.a.dz (H_i - H) = G. \lambda.dH \text{.....8}$$

3) Heat transfer from the liquid to the interface :-

$$h_L.a.dz (T_L - T_i) = L.C_L.dT_L \text{ .....9a}$$

where  $h_L$  Individual heat transfer coefficient in the liquid phase.

$T_L$  Bulk liquid temperature.

$T_i$  Liquid temperature at the interface.

It will assume that the area for heat transfer is equal to that available for mass transfer, though it may be somewhat greater when the packing is not completely wetted.

Rearranging equation (9a) :-

$$\frac{dT_L}{T_L - T_i} = \frac{h_L a}{L C_L} dz \quad \dots\dots\dots 9b$$

4) Heat transfer from the interface to the bulk of the gas.

$$h_G a \cdot dz (T_i - T_G) = G \cdot S \cdot dT_G \dots\dots\dots 10a$$

where  $h_G$  individual heat transfer coefficient in the gas phase.

Rearranging equation (10a) :-

$$\frac{dT_G}{T_i - T_G} = \frac{h_G a}{G S} dz \quad \dots\dots\dots 10b$$

From Lewis relation  $\frac{h}{h_D \rho} = S \dots\dots h = h_G = h_D \rho S \dots\dots$  sub into equation 9a :-

$$h_D \rho \cdot S \cdot a \cdot dz (T_i - T_G) = G \cdot S \cdot dT_G \dots\dots\dots 11$$

adding equations (8) and (11) :-

$$h_D \rho \cdot \lambda \cdot a \cdot dz (H_i - H) = G \cdot \lambda \cdot dH \dots\dots\dots 8$$

$$h_D \rho \cdot S \cdot a \cdot dz (T_i - T_G) = G \cdot S \cdot dT_G \dots\dots\dots 11$$

$$h_D \rho \cdot a \cdot dz [\lambda (H_i - H) + S (T_i - T_G)] dz = G (\lambda \cdot dH + S \cdot dT_G) \dots\dots\dots 12$$

$$h_D \rho \cdot a \cdot dz [(S T_i + \lambda H_i) - (S T_G + \lambda H)] dz = G (\lambda \cdot dH + S \cdot dT_G) \dots\dots\dots 13$$

Since ;  $H_G = S(T_G - T_o) + H \lambda$  in the bulk gas ....(  $dH_G = S dT_G + \lambda dH$  )

$H_i = S(T_i - T_o) + H_i \lambda$  at the interface

$$H_i - H_G = S(T_i - T_o) + H_i \lambda - S(T_G - T_o) - H \lambda \dots\dots\dots 14$$

$$H_i - H_G = (S T_i + H_i \lambda) - (S T_G + H \lambda) \dots\dots\dots 15$$

Sub equation (15) and into (13) :-

$$h_D \rho \cdot a \cdot dz (H_i - H_G) = G \cdot dH_G \dots\dots\dots 16$$

$$\therefore \frac{dH}{(H_i - H_G)} = \frac{h_D \rho a}{G} dz \dots\dots\dots 17$$

The height of packing in the cooling tower can be by integrating the above equation :-

$$\frac{h_D \rho a}{G} \int_0^z dz = \int_{H_{G1}}^{H_{G2}} \frac{dH}{(H_i - H_G)}$$

$$z = \frac{G}{h_D \rho a} \int_{H_{G1}}^{H_{G2}} \frac{dH}{(H_i - H_G)} \dots\dots\dots 18$$

$$z = HTU \cdot NTU$$

= Number of transfer unit \* Height of transfer unit

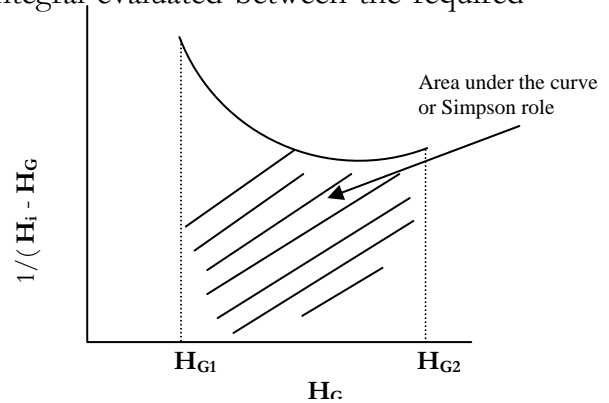


The mass transfer coefficient should be known or has to be estimated from suitable correlations. In order to determine the NTU, the integral of equation is to be evaluated. The conditions at the interface are not necessarily constant, and no direct relation is available between the enthalpy of the bulk gas  $H_G$  and that at the interface  $H_i$ . Numerical or approximate method or graphical evaluation of the integral must be done. It needs to know the values of  $H_i$  for a set of values of  $H_G$  within the range of enthalpy values given by the operating line ( $H_{G1} \leq H_G \leq H_{G2}$ ). The values of  $H_i$  can be obtained by the following procedure later.

- $1/(H_i - H_G)$  plotted vs.  $H_G$  and the integral evaluated between the required limits.

$H_{G1}$  enthalpy of the inlet gas.

$H_{G2}$  enthalpy of the outlet gas.



- **Approximate method of evaluating the integral of the height of tower.**

In this method it is assumed that the enthalpy difference ( $\Delta H = H_i - H_G$ ) varies in a parabolic manner. The three points taken to define the parabola are at the bottom and top of the column ( $\Delta H_1$  and  $\Delta H_2$ ) and  $\Delta H_m$  the value at the mean temperature in the column. The effective mean driving force is  $f \Delta H_m$ , where  $f$  is a factor for converting the driving force at the mean water temperature to the effective value which can be estimated from figure(10).

$$\Delta H_1 = H_i - H_{G1}$$

$$\Delta H_2 = H_i - H_{G2}$$

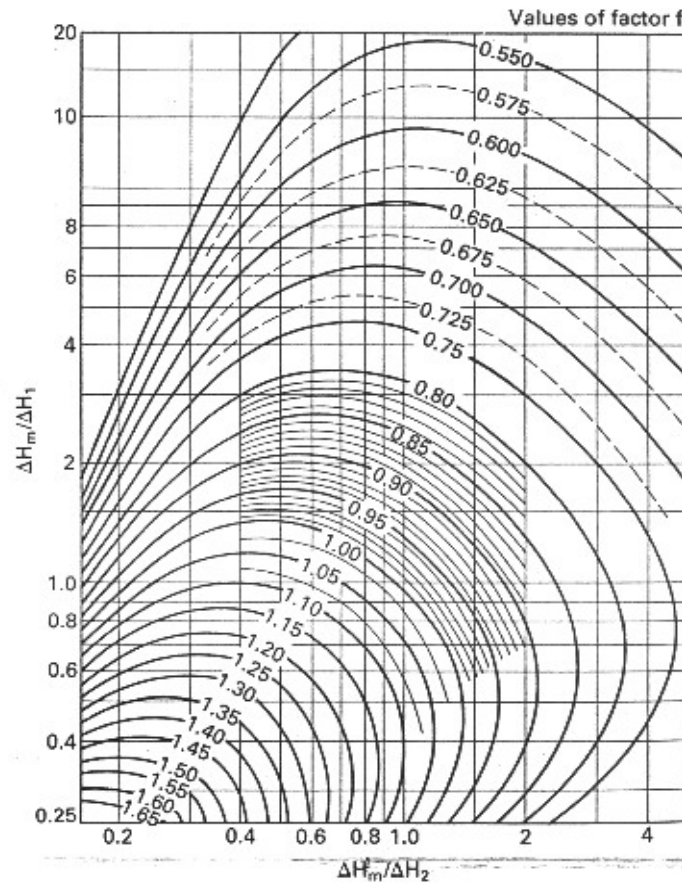
$$\Delta H_m = H_i - H_{Gmean}$$

$$\text{Mean temperature} = (T_{L1} + T_{L2})/2$$

$H_{Gmean}$  can be estimated at mean temperature.

$\Delta H_m/\Delta H_1$  plotted vs.  $\Delta H_m/\Delta H_2$  as shown in figure(10). The contours representing constant values of  $f$  are included. The height of tower is :-

$$z = \frac{G}{h_D \rho a} \frac{(H_{G2} - H_{G1})}{f \Delta H_m} \dots\dots\dots 19$$



Figure(10)

### Step by Step Procedure:-

- Suppose water is to be cooled at a mass rate  $L$  per unit area from a temperature  $T_{L2}$  to  $T_{L1}$ . The air at the inlet point of the at the bottom of the tower ( $G$   $\text{kg}/\text{m}^2.\text{s}$ ) have a temperature  $T_{G1}$ , humidity  $H_1$  and enthalpy  $H_{G1}$  which can be calculated from temperature - humidity chart (2) or from equation(10).
- The **operating line** AB is drawn using equation(6) which is a straight line of slope  $\frac{L}{G}$ , the operating line pass through the points A( $T_{L1}, H_{G1}$ ), and B( $T_{L2}, H_{G2}$ ). Point B corresponds to conditions at the top of the tower and the ordinate gives the enthalpy of the air leaving the column.
- The enthalpy –Temperature curve PQ ( saturation or equilibrium curve) for saturated air is plotted either using calculated data or from the humidity – enthalpy chart (4). The region below this line related to unsaturated air and the region above it is supersaturated air. If it is assumed that the air in contact with the liquid surface is saturated with vapor, this curve represents the relation between air enthalpy and temperature at the interface.

- The relation between liquid temperature, air enthalpy and conditions at the interface for any position in the tower is obtained by combining equations (9a)

and (16) :- 
$$\frac{H_G - H_i}{T_i - T_i} = -\frac{h_L}{h_D \rho} \dots\dots\dots*$$

and represented by a straight line of slope  $-(h_L/h_D \rho)$ .

**Estimation the leaving air temperature :-**

- The line for the bottom of the column passes through the point A( $T_{L1}, H_{G1}$ ) and cuts the enthalpy –temperature curve for saturated air at the point C [ the conditions at the interface ].

- The enthalpy and temperature of the air at the bottom of the tower are known ; the condition of the air can be represented by point E with coordinates ( $T_{G1}, H_{G1}$ ). The line AC is parallel to the temperature axis.

- The line EC is a straight line of slope  $dH_G/dT_G$  ;from equation (5),(11),(16) :-

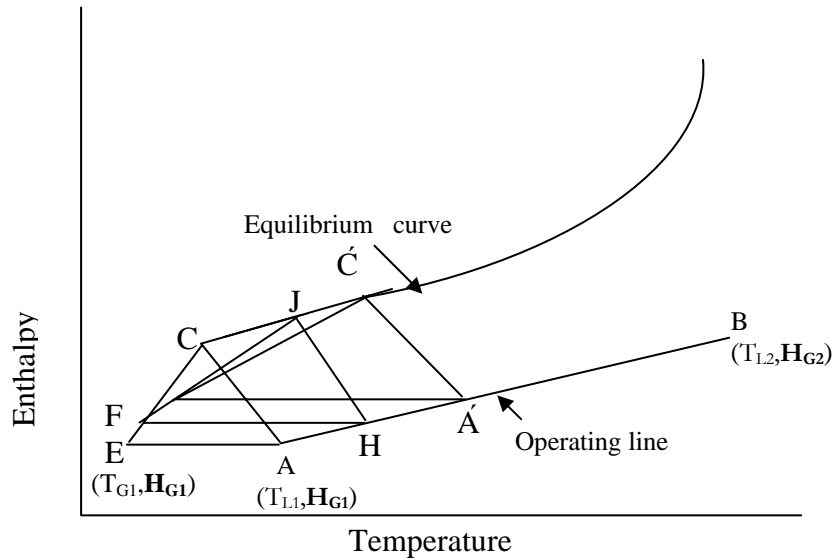
$$\frac{H_G - H_i}{T_G - T_i} = \frac{dH}{dT_G} \dots\dots\dots**$$

Which represents the rate of change of air enthalpy with air temperature at the bottom of the tower.

- The point F on EC line, will represents the condition of the gas at a small distance from the bottom. Liquid temperature is found by drawing through F a line parallel to the temperature axis. This cuts the operating line at point H, which indicates the liquid temperature.

- The temperature and enthalpy of the gas at the interface is obtained by drawing a line through H, parallel to AC (which is a tie line of slope  $-(h_L/h_D \rho)$ ). This line cuts the curve for saturated air at a point J, which represents the conditions of the gas at the interface. The rate of change of enthalpy with temperature for the gas is then given by the slope of the line FJ.

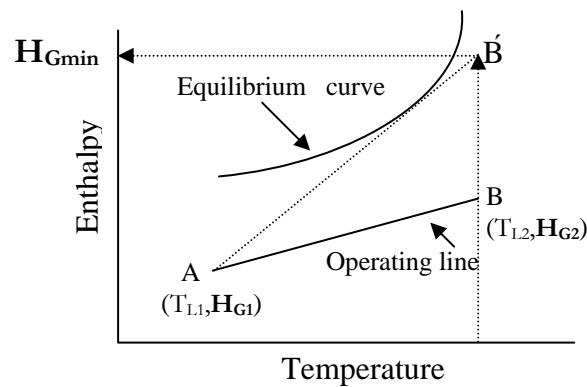
- The procedure is repeated until the curve representing the condition of the gas has been extended to a point whose ordinate is equal to the enthalpy of the gas at the top of the column. This point is obtained by drawing a straight line through B, parallel to the temperature axis. The final point on the line represents the condition of the air which leaves the top of the water- cooling tower.



Figure(11)

### 2.7.2 Minimum Gas (Air) Flow.

Often the air flow  $G$  is not fixed but must be set for the design of the cooling tower. The operating line is originally  $AB$  as shown in figure(12); but move toward the position of  $A'B'$ . As it does so, the column height would have to be increased to allow the required transfer to occur despite the smaller driving forces. Eventually, at the minimum vapor rate, an infinity tall column would be required. Even for such a tower, the outlet air would not be saturated at the liquid temperature. The situation is emphasized in figure(12) where the outlet gas-phase enthalpy is shown in point  $B$  for a finite column and at  $B'$  for infinite one.



Figure(12) Minimum gas rate.

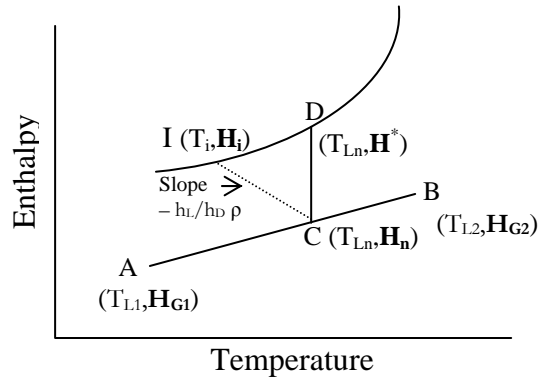
For minimum gas rate  $G_{\min}$ , the slope of the operating line become  $LC_L/G_{\min}$ ; and from point  $B'$  at  $T_{L2}$  estimate  $H_{G\min}$  as shown in the above figure then sub in operating line equation to calculate  $G_{\min}$ .

### 2.7.3 Overall Coefficients.

As shown in earlier (see equations(18) and (19)), the individual mass transfer coefficients is used and may be converted to overall coefficients if :-

- The equilibrium curve is straight line.
- The liquid phase resistance to heat transfer is very small compared to the gas phase resistance to mass transfer the actual interface temperature will approach the bulk temperature.

Therefore; the slope  $-h_L/h_D \rho$  approaches  $-\infty$  , and point I on figure (13) approaches the equilibrium equivalent of C located at point D.



Figure(13)

Therefore; equation(18) become :-

$$z = \frac{G}{h_D \rho a} \int_{H_{G1}}^{H_{G2}} \frac{dH}{(H^* - H_G)} \dots\dots\dots 20$$

### 2.7.4 Other Types of Towers.

For **cooling** or **humidification tower** the operating line lies below the equilibrium curve and water is cooled and air humidified. In a **dehumidification tower** cool water is used to reduced the humidity and temperature of the air that enters. In this case the operating line is above the equilibrium curve, and similar calculation method used. The height of tower for dehumidification tower is :-

For individual mass transfer coefficients:-

$$z = \frac{G}{h_D \rho a} \int_{H_{G1}}^{H_{G2}} \frac{dH}{(H_G - H^*)} \dots\dots\dots 21a$$

For overall mass transfer coefficients:-

$$z = \frac{G}{h_D \rho a} \int_{H_{G1}}^{H_{G2}} \frac{dH}{(H_G - H_i)} \dots\dots\dots 21b$$

### 2.7.5 Evaluation of Heat and Mass Transfer Coefficients.

Several workers have measured heat and mass transfer coefficients for water cooling towers and in humidifying towers :-

$$h_G a = 3.0 \ L^{0.26} G^{0.72}$$

$$h_L a = 1.04 * 10^4 \ L^{0.51} G$$

$$h_D a = 2.95 \ L^{0.26} G^{0.72}$$

Where L, G kg/m<sup>2</sup>.s ,h<sub>G</sub>a, h<sub>L</sub>a w/m<sup>3</sup>.k and h<sub>D</sub>a s<sup>-1</sup>

# Transport Phenomena

## CHAPTER 3

### *Evaporation*

#### **3.1 Introduction.**

Evaporation is one of the method used to concentrate an aqueous solution consisting of a non-volatile solute and a volatile solvent. Evaporation is achieved by adding heat to the solution to vaporize a portion of the solvent. The heat is supplied principally to provide the latent heat of vaporization.

**Various liquors which are to be evaporated may be classified as follows:**

- (a) Those which can be heated to high temperatures without decomposition, and those that can be heated only to a temperature of about 330 K "low temperature".
- (b) Those which yield solids on concentration, in which case crystal size and shape may be important, and those which do not.
- (c) Those which, at a given pressure, boil at about the same temperature as water, and those which have a much higher boiling point.

Evaporation differs from drying in that the residue is a liquid. Sometimes a highly viscous one rather than a solid; it differs from distillation in that the vapor usually is a single component, and even when the vapor as a mixture, no attempt is made in the evaporation step to separate the vapor into fractions; it differs from crystallization in that emphasis is placed on concentrating a solution rather forming and building crystals.

In a certain situation e.g. in the evaporation of brine to produce common salt, the line between evaporation and crystallization is far from sharp. Evaporation sometimes produce a slurry of crystal in a saturated mother liquor.

#### **3.2 Processing Factors.**

The selection of type of evaporator best suited for a particular service is governed by the characteristics of the feed and product Also depend on the pressure and temperature of the process. Points that must be considered are crystallization, salting and scaling, product quality, corrosion, and foaming. In the case of a **crystallizing evaporator**, the desirability of producing crystals of a definite uniform size usually limits the choice to evaporators having a positive means of circulation. **Salting**, which is the growth on body and heating-surface walls of a material having a solubility that increases with increase in temperature, is frequently encountered in crystallizing evaporators. It can be reduced or eliminated by keeping the evaporating liquid in close or frequent contact with a large surface area of crystallized solid. **Scaling** is the deposition and growth on

body walls, and especially on heating surfaces, of a material undergoing an irreversible chemical reaction in the evaporator or having a solubility that decreases with an increase in temperature. Scaling can be reduced or eliminated in the same general manner as salting. Both salting and scaling liquids are usually best handled in evaporators that do not depend on boiling to induce circulation. **Fouling** is the formation of deposits other than salt or scale and may be due to corrosion, solid matter entering with the feed, or deposits formed by the condensing vapor. **Product Quality** Considerations of product quality may require low holdup time and low-temperature operation to avoid thermal degradation. The low holdup time eliminates some types of evaporators, and some types are also eliminated because of poor heat-transfer characteristics at low temperature. Product quality may also dictate special materials of construction to avoid metallic contamination or a catalytic effect on decomposition of the product. **Corrosion** may also influence evaporator selection, since the advantages of evaporators having high heat-transfer coefficients are more apparent when expensive materials of construction are indicated. Corrosion and erosion are frequently more severe in evaporators than in other types of equipment because of the high liquid and vapor velocities used, the frequent presence of solids in suspension, and the necessary concentration differences.

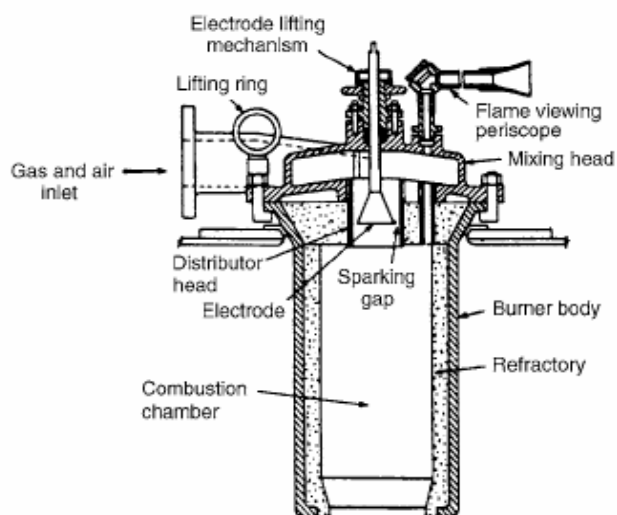
### 3.3 Evaporator Types and Applications.

The type of evaporators depend primarily on configuration of the heat transfer surface and on the means employed to provide agitation or circulation of the liquid. The heat added to a solution to vaporize the solvent may be either direct (representing by solar evaporation and by submerged combustion of a fuel) or indirect heating (representing by condensation of steam passes through the heating surface of the evaporator). Figure (1 and 2) show the type of evaporators.

The general types are :-

#### ❖ Evaporators with direct heating

The submerged combustion of a gas, such as natural gas, has been used for the concentration of very corrosive liquors, including spent pickle liquors, weak phosphoric and sulfuric acids. A suitable burner for direct immersion in the liquor, as shown in Figure(1).

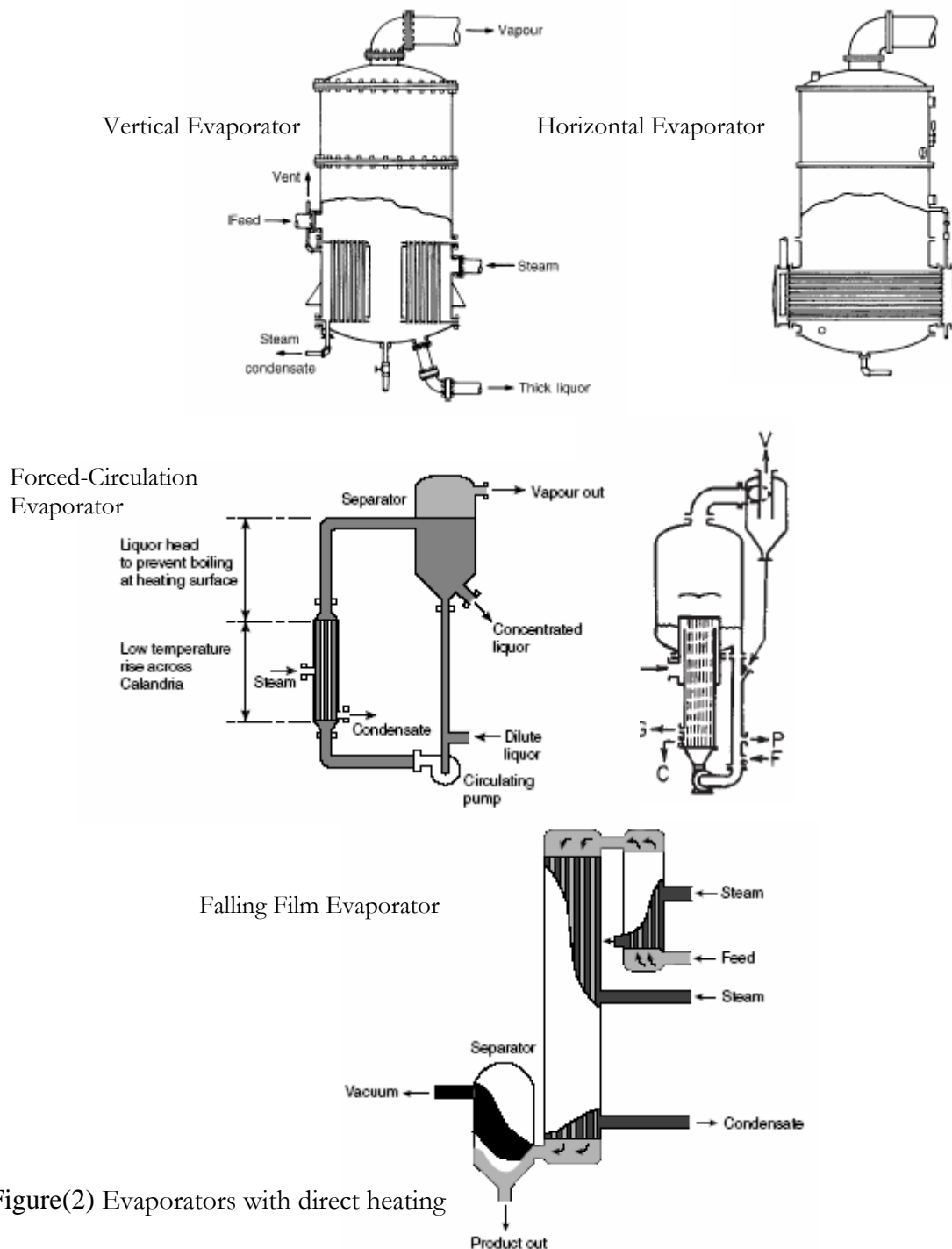


Figure(1) Burner for submerged combustion



❖ **Evaporators with direct heating**

- **Natural circulation evaporators :-**
  - **Short-Tube Vertical Evaporators** (Clear liquids ,crystalline product if propeller is used, relatively non-corrosive liquids and mild scaling solutions)
  - **Long-Tube Vertical Evaporators**(used widely for milk production ,Foaming liquids and Corrosive solutions).
  - **Horizontal-Tube Evaporators**(used primarily for seawater evaporation)
- **Forced-Circulation Evaporators** (useful for highly viscous ,corrosive solutions Crystalline product).
- **Falling-film**(useful for sensitive material such as orange juice).
- **Agitated thin-film evaporator**(useful for highly viscous and sensitive viscous solution)



Figure(2) Evaporators with direct heating

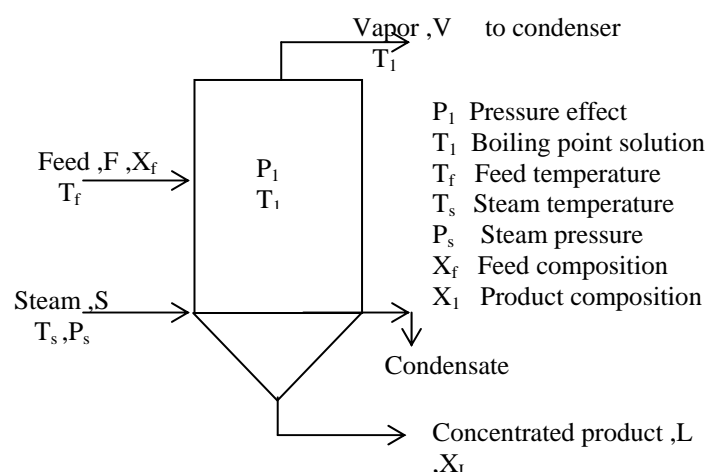
### 3.4 Evaporator Arrangement.

#### 3.4.1 Single Effect Evaporator.

Single-effect evaporators are used when :-

- The throughput is low.
- When a cheap supply of steam is available.
- When expensive materials of construction must be used as is the case with corrosive feedstock and when the vapor is so contaminated so that it cannot be reused.

Single effect units may be operated in batch, semi-batch or continuous batch modes or continuously.



Figure(3) Single effect Evaporator

\* In a single effect evaporator, the condensate product and the solution in the evaporator have the same composition and temperature  $T_1$  (Boiling point of solution).

\* The temperature of vapor is also  $T_1$  since it is in equilibrium with boiling solution.

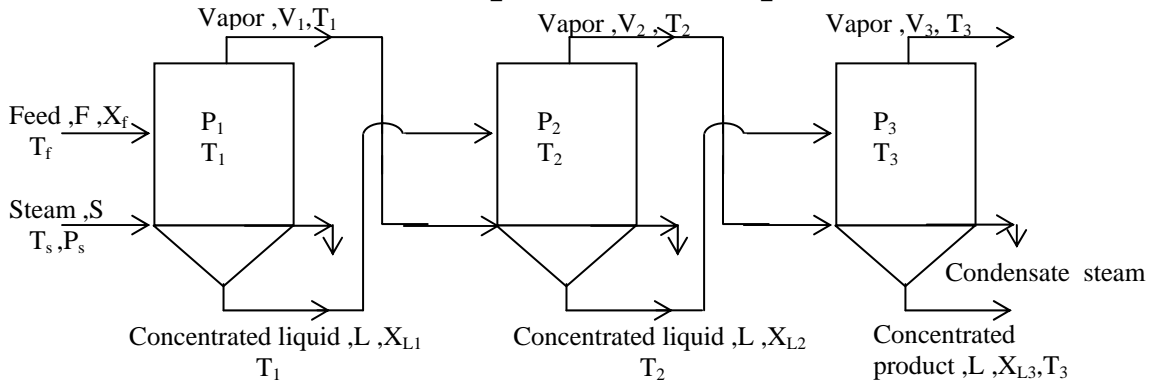
\* The pressure is  $P_1$ , which is the vapor pressure of the solution at  $T_1$ .

\*The solution in evaporator is assumed to be completely mixed.

### 3.4.2 Multiple Effect Evaporators.

Multiple effect evaporator is the principle means in use for economizing consumption, and used when the required capacity is large.

#### i. Forward Feed Multiple Effect Evaporators.



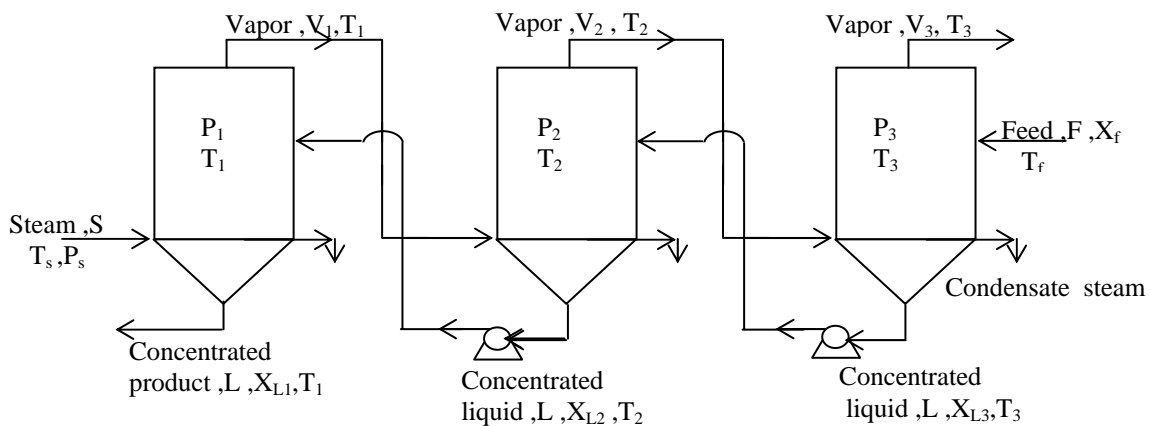
Figure(4) Forward Feed Evaporators

$$P_1 > P_2 > P_3$$

$$T_1 > T_2 > T_3$$

In this type of operation, the feed is introduced in the first effect and passed from first to the next effect parallel to the vapor flow. Product is withdrawn from the last effect. This method of evaporation is used when the feed is hot or when the concentration product would be damaged or would be deposited scale at high temperature. In forward feed operation, the liquor can be transferred by pressure difference alone, thus dominating all intermediate liquid pumps, i.e. if the effect at  $P_1 = 1$  atm the last effect ( $P_3$ ) will be under vacuum.

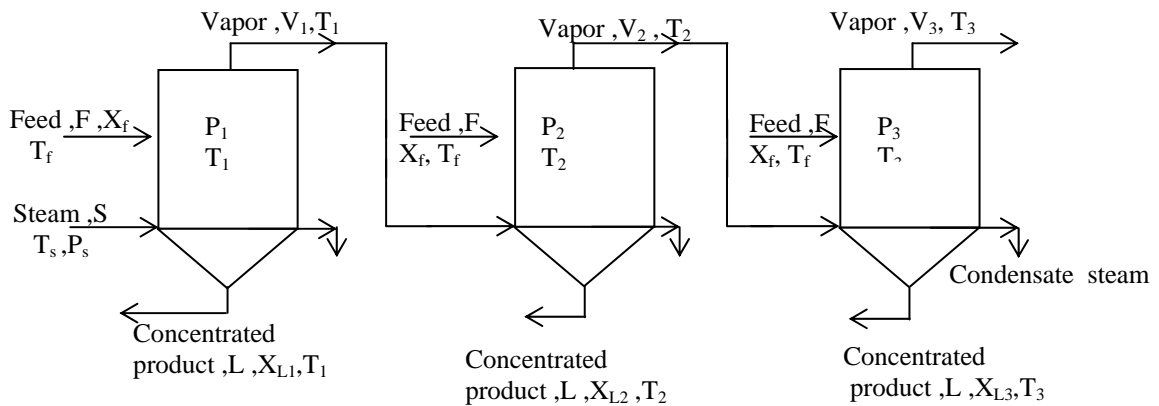
#### ii. Backward Feed Multiple Effect Evaporators.



Figure(5) Backward Feed Evaporators

In backward feed operation, the raw feed enters the last effect; the discharge from this effect becomes the feed to the next and so on until product is discharged from the first effect. This operation is used when the feed is cold since much less liquid must be heated to higher temperature and also used when the product is so viscous that high temperatures are needed to keep the viscosity low enough to give reasonable heat transfer coefficients. Liquid pumps are used in each effect, since the flow from low to high pressure.

### iii. Parallel Feed Multiple Effect Evaporators.



Figure(6) Parallel Feed Evaporators

In parallel feed evaporator, involves the introduction of raw feed and withdrawal of concentrated product at each effect of the evaporator. It is used primarily when the feed is substantially saturated and the product is solid; as the evaporation of brine to make common salt.

### 3.5 Heat Transfer in Evaporator.

Heat transfer is the most important single factor in evaporator design, since the heating surface represents the largest part of evaporator cost. The type of evaporator selected is the one having the highest heat transfer coefficient.

The rate of heat transfer is expressed as :-

$$Q = U.A.\Delta T \dots\dots\dots 1$$

Heat transfer coefficient is a strong function of temperature difference  $\Delta T$ .

Where U overall heat transfer coefficient.

A heat transfer area.

$\Delta T$  temperature difference between the heating medium and the boiling liquor ( $T_{\text{effect}} - T_{\text{solution}}$ ).

The temperature of boiling liquor depends on :-

- i. Pressure in the evaporator.
- ii. Solute concentration.
- iii. Liquid head (also called hydrostatic head).

When the solution has the characteristics of pure water, its boiling point can not be read from steam tables .

#### 3.5.1 Boiling point Rise and Hydrostatic head.

**Boiling Point Rise** :- At a given pressure the boiling point of the solution is higher than that of pure water. The increase in boiling point over that of water is known **Boiling Point Rise (BPR)** of solution. The boiling point rise must be subtracted from the temperature difference that is predicted from steam table.

$$\Delta T = T_{\text{effect}} - (T_{\text{pure water}} + \text{BPR})$$

$$T_{\text{solution}} = T_{\text{pure water}} + \text{BPR}$$

The boiling point rise for common solution can be estimated from Dühring chart by using the boiling point of pure water at a given pressure  $P_1$  and the **concentration of product** solution as shown in figure(7a and b)

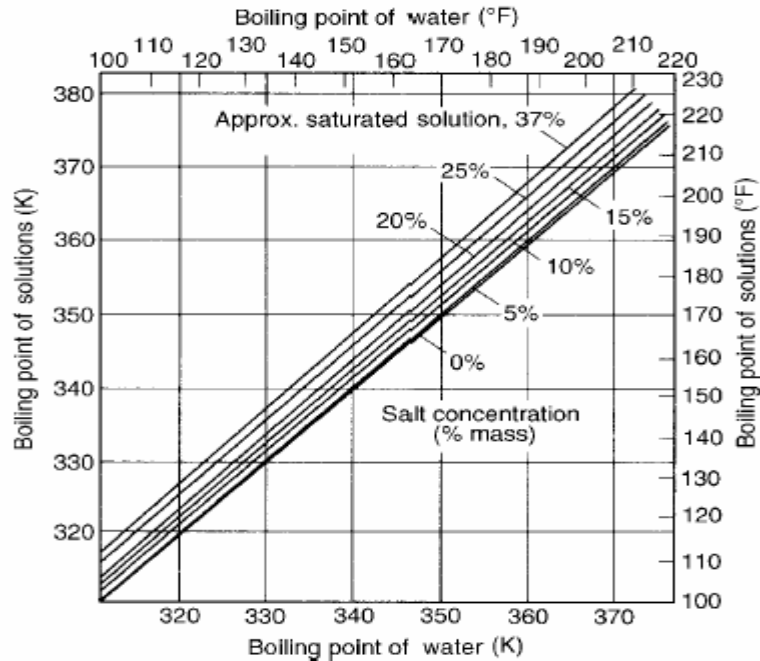


Figure (7a) Boiling point of solutions of sodium chloride as a function of the boiling point of water Dühring lines.

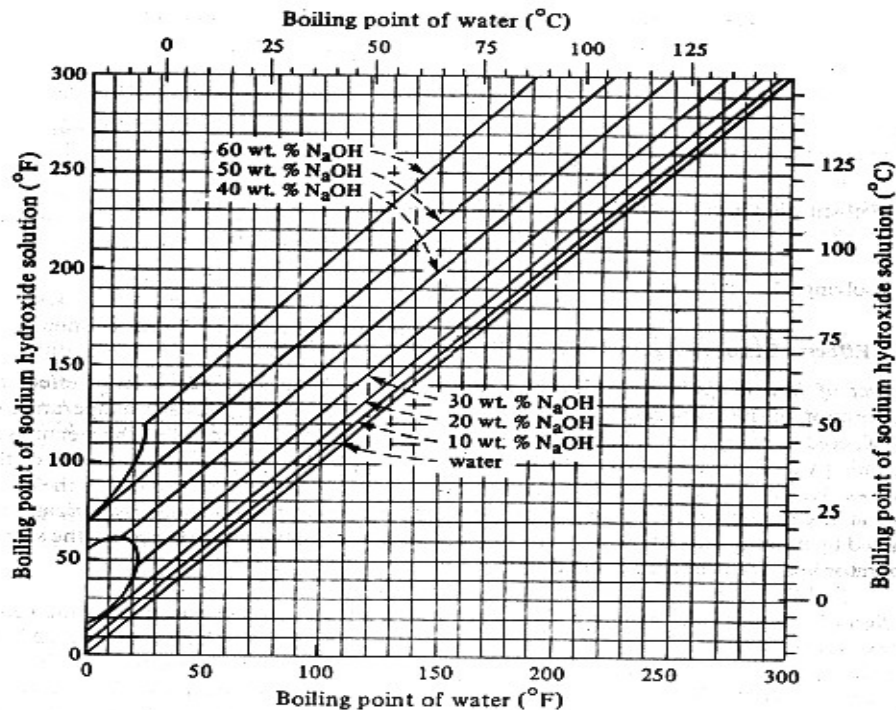


Figure (7b) Boiling point of solutions of sodium hydroxide as a function of the boiling point of water Dühring lines.

**Hydrostatic Head**:- The effect of liquid head above the tube bundle is to add to pressure and rise the boiling temperature

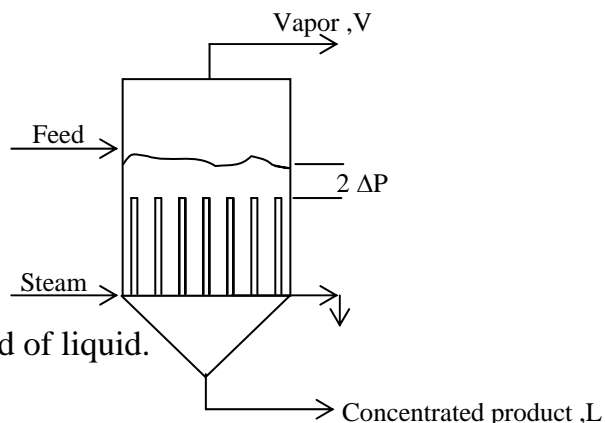
$$\Delta P = \text{hydrostatic head (liquid head)} \\ = (1/2) \text{ actual liquid head}$$

$$\dot{P} = \rho \cdot g \cdot h$$

Where  $\rho$  density of solution

$h$  liquid head (level)

$\dot{P}$  pressure due to hydrostatic head of liquid.



### 3.5.2 Economy and Capacity.

**Economy** is defined as the amount evaporated per unit amount of steam used; which it depends on the number of effect and influenced by the temperature of the feed.

$$\text{Economy} = \frac{V}{S}$$

**Capacity** is defined as the number of pounds of water vaporized per hour, which is directly proportional to :-

- The difference between the condensing temperature of the steam supplied and the temperature of the boiling solution in effect (in case of multiple effect it is the last one).
- The overall coefficient of heat transfer from steam to solution.

$$\text{Capacity} = V$$

## 3.6 Calculation Method.

### 3.6.1 Single Effect Evaporator.

#### 1) Material Balance.

Overall M.B.

$$F = L + V \dots\dots\dots 2$$

Where  $F$  liquor feed rate (mass/time).

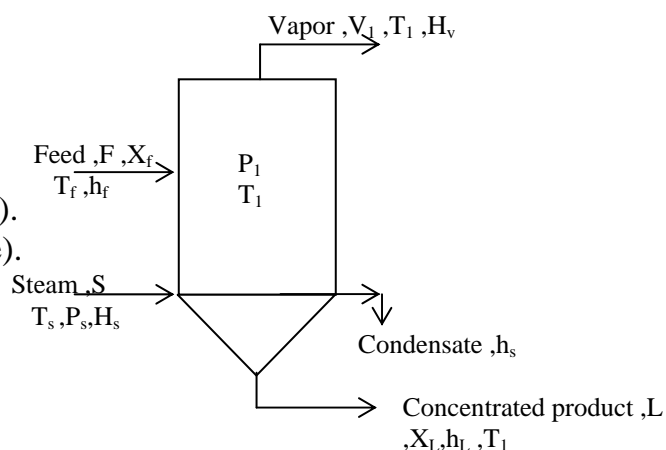
$V$  evaporation rate (mass/time).

$L$  product (concentrate) rate (mass/time).

Solute Balance

$$F \cdot X_f = L \cdot X_L \dots\dots\dots 3$$

Where  $X_f$ ,  $X_L$  are the feed and product concentration respectively with respect to solute.



## 2) Heat Balance

$$Fh_f + SH_s = VH_v + Sh_s + Lh_L$$

Where  $\lambda_s = H_s - h_s$  .....for saturated steam used.

$$\therefore Fh_f + S\lambda_s = VH_v + Lh_L \dots\dots\dots 4$$

$$S\lambda_s = Q = UA\Delta T \dots\dots\dots 5$$

Where  $h_f$  and  $h_L$  enthalpy of feed and product (this depend on solute concentration and temperature )kJ/kg.

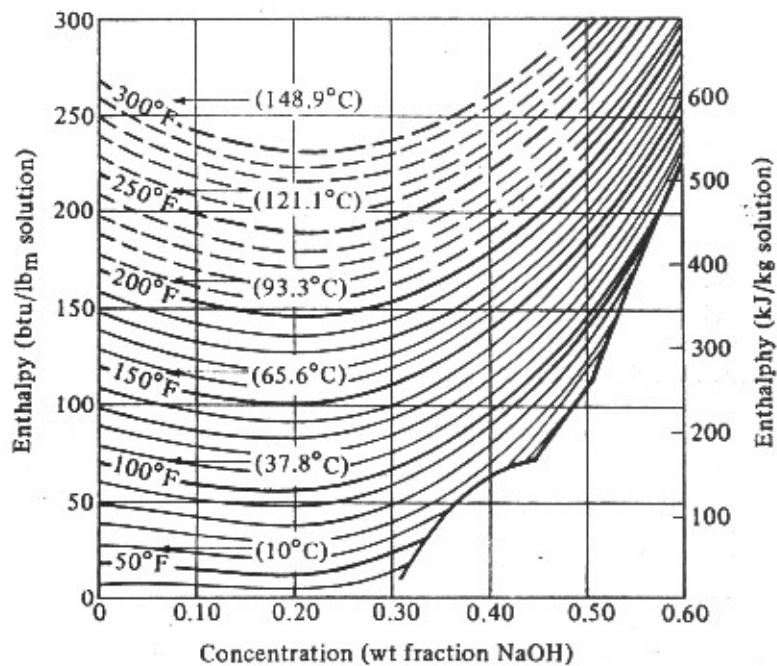
$\lambda_s$  latent heat of steam kJ/kg .

$H_v$  enthalpy of vapor kJ/kg.

$h_s$  enthalpy of condense steam kJ/kg.

Enthalpy of feed  $h_f$  and product  $h_L$  can be estimated from:-

- $h = C_p \Delta T$
- Dühring chart see figure(8).



Figure(8) Enthalpy –Temperature Dühring chart.

**Example(1)**

A single effect evaporator is used to concentrate 7kg/s of a solution from 10 to 50wt% of solid. Steam is available at 205kN/m<sup>2</sup>, and evaporation takes place at 13.5kN/m<sup>2</sup>, if the overall heat transfer coefficient is 3205kW/m<sup>2</sup>.k. Calculate the heating surface required and the amount of steam used if the feed to the evaporator is at 294k.

The specific heat of solution is :-

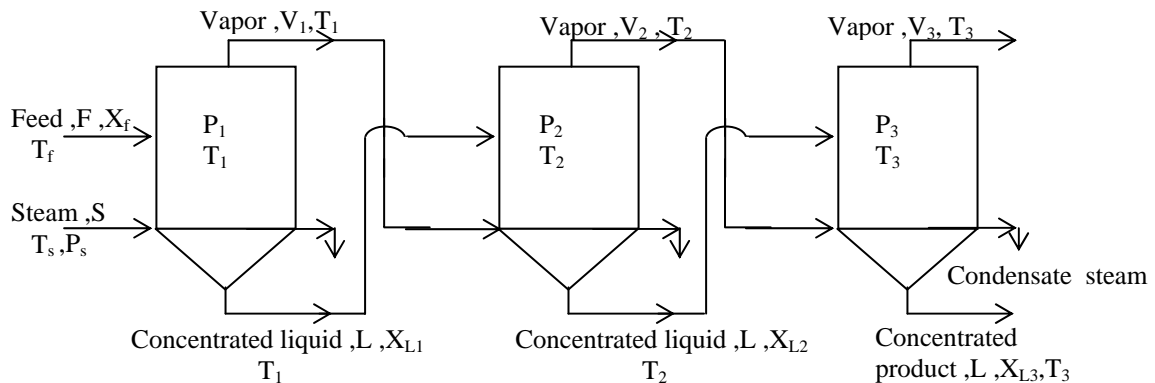
10% solution = 3.7kJ/kg.k

50% solution = 3.14kJ/kg.k

**Example(2)**

A single effect evaporator is used to concentrate 5000 kg/h of solution of sodium hydroxide from 10% to 25% solids. Steam is available at 143.27KPa absolute; the vapor space is maintained at 62.745KPa gage pressure. The feed enters at its boiling point corresponding to the vapor space pressure. Calculate: 1.The steam consumption per hour.

2.If the available heat transfer area is 35m<sup>2</sup> estimate the heat transfer coefficient.

**3.6.2 Multiple Effect Evaporator.****Forward feed****1) Material Balance.**

Overall M.B.

$$F = L_3 + V \dots\dots\dots 2$$

Solute Balance

$$F.X_f = L.X_{L3} \dots\dots\dots 3$$

**2) Material Balance for Each Effect.**

$$\left. \begin{array}{l} \text{1st effect } F = V_1 + L_1 \\ \text{2nd effect } L_1 = V_2 + L_2 \\ \text{3rd effect } L_2 = V_3 + L_3 \end{array} \right\} \dots\dots\dots 6$$



$$V = V_1 + V_2 + V_3 \dots\dots\dots 7$$

### 3) Heat Balance for Each Effect.

#### 1<sup>st</sup> effect

$$Fh_f + S\lambda_s = V_1H_{v_1} + L_1h_{L_1} \dots\dots\dots 8$$

$$h_f = Cp_f(T_f - T_{ref}) \quad , h_{L_1} = Cp_{L_1}(T_1 - T_{ref})$$

or from Dühring chart  $h_f$  at  $X_f$  and  $T_f$   
 $h_{L,1}$  at  $X_{L,1}$  and  $T_1$

#### 2<sup>nd</sup> effect

$$L_1h_{L_1} + V_1\lambda_1 = V_2H_{v_2} + L_2h_{L_2} \dots\dots\dots 9$$

$$h_{L_1} = Cp_{L_1}(T_1 - T_{ref}) \quad , h_{L_2} = Cp_{L_2}(T_2 - T_{ref})$$

or from Dühring chart  $h_{L,1}$  at  $X_{L,1}$  and  $T_1$   
 $h_{L,2}$  at  $X_{L,2}$  and  $T_2$

#### 3<sup>rd</sup> effect

$$L_2h_{L_2} + V_2\lambda_2 = V_3H_{v_3} + L_3h_{L_3} \dots\dots\dots 10$$

$$h_{L_2} = Cp_{L_2}(T_2 - T_{ref}) \quad , h_{L_3} = Cp_{L_3}(T_3 - T_{ref})$$

or from Dühring chart  $h_{L,2}$  at  $X_{L,2}$  and  $T_2$   
 $h_{L,3}$  at  $X_{L,3}$  and  $T_3$

$\lambda_1, \lambda_2, \lambda_3$  from steam table at  $P_1, P_2, P_3$  respectively.

### Calculation Steps

Considering for three evaporators arranged as shown in the above figure, in which the temperatures and pressures are  $T_1, T_2, T_3$ , and  $P_1, P_2, P_3$ , respectively, in each unit, then the heat transmitted per unit time across each effect is:

$$\text{Effect 1} \quad Q_1 = U_1A_1\Delta T_1, \quad \text{where } \Delta T_1 = (T_s - T_1),$$

$$\text{Effect 2} \quad Q_2 = U_2A_2\Delta T_2, \quad \text{where } \Delta T_2 = (T_1 - T_2),$$

$$\text{Effect 3} \quad Q_3 = U_3A_3\Delta T_3, \quad \text{where } \Delta T_3 = (T_2 - T_3).$$

- If the liquor **has no** boiling point rise, and neglect the effect of hydrostatic head, therefore;

$$\Delta T_1 = (T_s - T_1)$$

$$T_1 = T_1' \text{ boiling point of pure water at } P_1 \text{ (operating pressure)}$$

$$\text{This is also for } \Delta T_2 \text{ and } \Delta T_3, \quad \Delta T_2 = (T_1' - T_2) \quad , \quad \Delta T_3 = (T_2' - T_3)$$

- If the liquor **has** boiling point rise then :-

$$\Delta T_1 = (T_s - T_1)$$

$$T_1 = T'_1 + \text{BPR} \quad \text{or } T_1 \text{ from Dühring chart}$$

This is also for  $\Delta T_2$  and  $\Delta T_3$ .

The heat required  $Q_1$  transfer across  $A_1$  appear as latent heat in the vapor  $V_1$  and is used as steam in the second effect and so on.

Hence,  $Q_1 = Q_2 = Q_3 = Q$ .....11

$$U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2 = U_3 A_3 \Delta T_3 \dots\dots\dots 12$$

Where  $A_1 = A_2 = A_3$

Therefore;  $U_1 \Delta T_1 = U_2 \Delta T_2 = U_3 \Delta T_3 \dots\dots\dots 13$

$$\therefore \frac{Q}{A} = U_1 \Delta T_1 = U_2 \Delta T_2 = U_3 \Delta T_3 \dots\dots\dots 14$$

On this analysis, the difference in temperature across each effect is inversely proportional to the heat transfer coefficient.

$$\Sigma \Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 = T_s - T_3 \dots\dots\dots 15$$

$$\Delta T_1 = \Sigma \Delta T \left[ \frac{\frac{1}{U_1}}{\frac{1}{U_1} + \frac{1}{U_2} + \frac{1}{U_3}} \right] \dots\dots\dots 16$$

$$\Delta T_2 = \Sigma \Delta T \left[ \frac{\frac{1}{U_2}}{\frac{1}{U_1} + \frac{1}{U_2} + \frac{1}{U_3}} \right] \dots\dots\dots 17$$

$$\Delta T_3 = \Sigma \Delta T \left[ \frac{\frac{1}{U_3}}{\frac{1}{U_1} + \frac{1}{U_2} + \frac{1}{U_3}} \right] = \Sigma \Delta T - (\Delta T_2 + \Delta T_1) \dots\dots\dots 18$$

The water evaporated in each effect is proportional to  $Q$ , so to estimate the capacity of three effect by adding the value of  $Q$  for each effect :-

$$Q = Q_1 + Q_2 + Q_3 \dots\dots\dots 19$$

$$= U_1 A_1 \Delta T_1 + U_2 A_2 \Delta T_2 + U_3 A_3 \Delta T_3 \dots\dots\dots 20$$

Assume  $U$  are equal for each effect and  $A$  thus :-

$$Q = U A (\Delta T_1 + \Delta T_2 + \Delta T_3) \dots\dots\dots 21$$

$$= UA\Delta T.....22$$

### **Procedure Steps :-**

1. The outlet concentration and pressure of the last effect are known, if BPR present, this can be determined from relation given or Dühring chart.
2. Determine the total amount evaporated by overall material balance and then calculated the concentration for each effect.
3. Estimate the temperature drops  $\Delta T_1, \Delta T_2, \Delta T_3$  from equations 16,17,18 .
4. Calculate the amount vaporize , the concentration of liquid and value of Q transferred in each effect from material balance and heat balance, if the amount vaporize differ appreciably from those in step 2 then step 2,3,4 repeated using amount of evaporation calculated in step 4.
5. Using equation  $Q = UA\Delta T$  to calculate  $A_1, A_2, A_3$  if the area are not equal , then step6.
6. calculate the average value of A as follows :-  $A_m = A = (A_1 + A_2 + A_3)/3$  then calculat4 a new values of  $\Delta T'_1, \Delta T'_2, \Delta T'_3$  :-

$$\Delta T'_1 = \frac{\Delta T_1 A_1}{A_m}, \quad \Delta T'_2 = \frac{\Delta T_2 A_2}{A_m}, \quad \Delta T'_3 = \frac{\Delta T_3 A_3}{A_m}$$

$$\Sigma \Delta T' = \Delta T'_1 + \Delta T'_2 + \Delta T'_3$$

7. Use the new  $\Delta T'$  from step 6 in the calculation. The calculation with step 4 will continued until the area are equal. If the BPR is present then using the concentration from step 4 to determine the BPR and boiling point in each effect, this gives a new value of summation of  $\Delta T'$  by subtracting the summation of all BPR from the overall  $\Delta T'$ . Calculate new value of  $\Delta T'$  from equation  $Q = UA\Delta T'$

$$\Sigma BPR = BPR_1 + BPR_2 + BPR_3$$

$$\Delta T' = \Sigma \Delta T' - \Sigma BPR$$

### Sheet NO.3 - Evaporator.

- Q1.** An evaporator is concentrating  $F$  kg/h at 311K of a 20wt% solution of NaOH to 50wt %. The saturated steam used for heating is at 399.3K. The pressure in the vapor space of the evaporator is 13.3 KPa abs. The overall coefficient is  $1420 \text{ W/m}^2\cdot\text{K}$  and the area is  $86.4 \text{ m}^2$ . Calculate the feed rate  $F$  of the evaporator.
- Q2.** Determine the heating surface area required for the production of  $2.5 \text{ kg/s}$  of 50wt% NaOH solution from 15 wt% NaOH feed solution which entering at  $100^\circ\text{C}$  to a single effect evaporator. The steam is available as saturated at 451.5K and the boiling point rise (boiling point elevation) of 50wt% solution is 35K. the overall heat transfer coefficient is  $2000 \text{ W/m}^2\cdot\text{K}$ . The pressure in the vapor space of the evaporator at atmospheric pressure. The solution has a specific heat of  $4.18 \text{ kJ/kg}\cdot\text{K}$ . The latent heat of vaporization under these condition is  $2257 \text{ kJ/kg}$ .
- Q3.** It is desired to design a double effect evaporators for concentrating a certain caustic soda solution from 12.5wt% to 40wt%. The feed at  $50^\circ\text{C}$  enters the first evaporator at a rate of  $2500 \text{ kg/h}$ . Steam at atmospheric pressure is being used for the said purpose. The second effect is operated under 600mmHg vacuum. If the overall heat transfer coefficients of the two stages are 1952 and  $1220 \text{ kcal/m}^2\cdot\text{h}\cdot^\circ\text{C}$ . respectively, determine the heat transfer area of each effect. The BPR will be considered and present for the both effect.
- Q4.** A forced circulation triple-effect evaporator using forward feed is to be used to concentrate a 10wt% NaOH solution entering at  $37.8^\circ\text{C}$  to 50wt%. The steam used enter at 58.6KPa gage. The absolute pressure in the vapor space of the third effect is 6.76KPa. The feed rate is  $13608 \text{ kg/h}$ , heat transfer coefficient are  $U_1=6246$ ,  $U_2=3407$  and  $U_3=2271 \text{ W/m}^2\cdot\text{K}$ . All effect have the same area, Calculate the surface area required and steam consumption.

مع أسئلة الكتاب :-

14,9,10,12,13,14,23,24,25,26.

# Transport Phenomena

## CHAPTER 4

### *Liquid Filtration*

#### 4.1 Introduction.

The separation of solids from a suspension in a liquid by means of a porous medium or screen which retains the solids and allows the liquid to pass is termed filtration. The separation of solids from a suspension in a liquid by means of a porous medium or screen which retains the solids and allows the liquid to pass is termed filtration. The particle is a separate phase or cake and the passes as the clear filtrate. In general, the pores of the medium are **larger** than the particles which are to be removed, and the filter works efficiently only after an initial deposit has been trapped in the medium. Filtration is essentially a mechanical operation and is less demanding in energy than evaporation or drying where the high latent heat of the liquid, which is usually water, has to be provided.

Volumes of the suspensions to be handled vary from the extremely large quantities involved in water purification and ore handling in the mining industry to relatively small quantities, as in the fine chemical industry where the variety of solids is considerable. In most industrial applications it is the solids that are required and their physical size and properties are of paramount importance.

The valuable product may be the clear filtrate from the filtration or the solid cake. In some cases complete removal of the solid particles is required and in other cases only partial removal.

Typical filtration operation is illustrated in Figure 7.1, which shows the filter medium, in this case a cloth, its support and the layer of solids, or filter cake, which has already formed. The cake gradually builds up on the medium and the resistance to flow progressively increases. During the initial period of flow, particles are deposited in the surface layers of the cloth to form the true filtering medium.

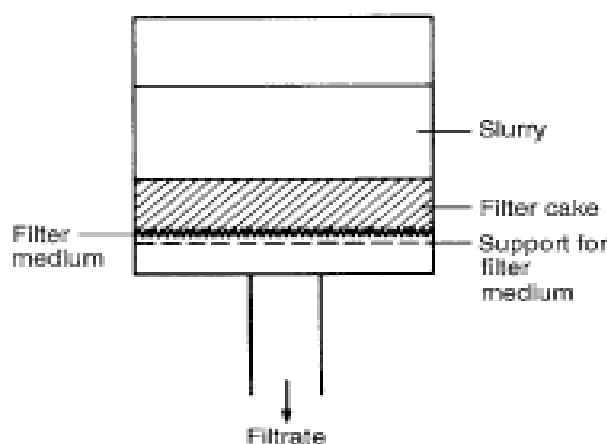


Figure 1 Principle of filtration

**The main factors to be considered when selecting equipment and operating conditions are:**

- (a) The properties of the fluid, particularly its viscosity, density and corrosive properties.
- (b) The nature of the solid—its particle size and shape, size distribution, and packing characteristics.
- (c) The concentration of solids in suspension.
- (d) The quantity of material to be handled, and its value.
- (e) Whether the valuable product is the solid, the fluid, or both.
- (f) Whether it is necessary to wash the filtered solids.
- (g) Whether very slight contamination caused by contact of the suspension or filtrate with the various components of the equipment is detrimental to the product.
- (h) Whether the feed liquor may be heated.
- (i) Whether any form of pretreatment might be helpful.

**The most important factors on which the rate of filtration then depends will be:**

- (a) The drop in pressure from the feed to the far side of the filter medium.
- (b) The area of the filtering surface.
- (c) The viscosity of the filtrate.
- (d) The resistance of the filter cake.
- (e) The resistance of the filter medium and initial layers of cake.

**Two basic types of filtration processes** may be identified, although there are cases where the two types appear to merge. In the **first**, frequently referred to as *cake filtration*, the particles from the suspension, which usually has a high proportion of solids, are deposited on the surface of a porous septum which should ideally offer only a small resistance to flow. As the solids build up on the septum, the initial layers form the effective filter medium, preventing the particles from embedding themselves in the filter cloth, and ensuring that a particle-free filtrate is obtained.

In the **second** type of filtration, *depth* or *deep-bed filtration*, the particles penetrate into the pores of the filter medium, where impacts between the particles and the surface of the medium are largely responsible for their removal and retention. This configuration is commonly used for the removal of fine particles from very dilute suspensions, where the recovery of the particles is not of primary importance. Typical examples here include air and water filtration.

## **4.2. Filter Media and Filter Aids.**

- ❖ **Filter Media:-**The filter media for the industrial filtration must fulfill a number of requirements. First and foremost, it must remove the solids to be filtered from the slurry and give a clear filtrate. Also the pores should not become plugged so that the rate of filtration becomes too slow. The

filter medium must allow the filter cake to be removed easily and clearly. Obviously; it must have sufficient strength to not tear and must be chemically resistance to the solutions used. Some widely used filter media are twill or duck weave heavy cloth, woven heavy cloth, glass, paper and nylon cloth.

- ❖ **Filter Aids:**-Certain filter aids may be used to aid filtration. Those are often incompressible diatomaceous earth or kieselguhr, which is composed primarily of silica. Also used are wood cellulose and other inert porous solids. These filter aids can be used in a number of ways. They can be used as a precoat before the slurry is filtered. This will prevent gelatinous type solids from plugging the filter medium and also give a clearer filtrate. They can also be added to the slurry before filtration. This increases the porosity of the cake and reduces resistance of the cake during filtration. The use of filter aids is usually limited to cases where the cake is discarded or to cases where the precipitate can be separated chemically from the filter aid.

### 4.3. Filtration Equipments.

The most suitable filter for any given operation is the one which will fulfill the requirements at minimum overall cost. Since the cost of the equipment is closely related to the filtering area, it is normally desirable to obtain a high overall rate of filtration. This involves the use of relatively high pressures although the maximum pressures are often limited by mechanical design considerations. Although a higher throughput from a given filtering surface is obtained from a continuous filter than from a batch operated filter, it may

sometimes be necessary to use a batch filter, particularly if the filter cake has a high resistance, since most continuous filters operate under reduced pressure and the maximum filtration pressure is therefore limited. Other features which are desirable in a filter include ease of discharge of the filter cake in a convenient physical form, and a method of observing the quality of the filtrate obtained from each section of the plant. These factors are important in considering the types of equipment available. The most common types are filter presses, leaf filters, and continuous rotary filters. In addition, there are filters for special purposes, such as bag filters, and the disc type of filter which is used for the removal of small quantities of solids from a fluid. The most important factors in filter selection are the specific resistance of the filter cake, the quantity to be filtered, and the solids concentration.

For free-filtering materials, a rotary vacuum filter is generally the most satisfactory since it has a very high capacity for its size and does not require any significant manual attention. If the cake has to be washed, the rotary drum is to be preferred to the rotary leaf. If a high degree of washing is required, however, it is usually desirable to re-pulp the filter cake and to filter a second time. For large-scale filtration, there are three principal cases where a rotary vacuum filter will not be used. Firstly, if the specific resistance is high, a positive pressure filter will be required, and a filter press may well be suitable, particularly if the solid content is not so high that frequent dismantling of the press is necessary. Secondly, when

efficient washing is required, a leaf filter is effective, because very thin cakes can be prepared and the risk of channeling during washing is reduced to a minimum. Finally, where only very small quantities of solids are present in the liquid, an edge filter may be employed.

#### 4.4 Basic Theory of Filtration.

The calculation of the rate of flow of a fluid through a bed of granular material, and these are now applied to the flow of filtrate through a filter cake. Some differences in general behavior may be expected, however, because the cases so far considered relate to uniform fixed beds, whereas in filtration the bed is steadily growing in thickness. There are two quite different methods of operating filter, if the filtration pressure is constant; the rate of flow progressively diminishes whereas, if the flow rate is to be maintained constant, the pressure must be gradually increased. Because the particles forming the cake are small and the flow through the bed is slow, streamline conditions are almost invariably obtained, and, at any instant, the flow rate of the filtrate may be represented by the following form of equation:-

$$u_c = \frac{1}{A} \frac{dV}{dt} = \frac{1}{5} \frac{e^3}{(1-e)^2} \frac{-\Delta P}{S^2 \mu l} \dots\dots\dots 1$$

Where  $V$  is the volume of filtrate which has passed in time  $t$ ,  $A$  is the total cross-sectional area of the filter cake,  $u_c$  is the superficial velocity of the filtrate,  $l$  is the cake thickness,  $S$  is the specific surface of the particles,  $e$  is the voidage,  $\mu$  is the viscosity of the filtrate, and  $P$  is the applied pressure difference.

In deriving this equation it is assumed that the cake is uniform and that the voidage is constant throughout. In the deposition of a filter cake this is unlikely to be the case and the voidage,  $e$  will depend on the nature of the support, including its geometry and surface structure, and on the rate of deposition. The initial stages in the formation of the cake are therefore of special importance for the following reasons:

- (a) For any filtration pressure, the rate of flow is greatest at the beginning of the process since the resistance is then a minimum.
- (b) High initial rates of filtration may result in plugging of the pores of the filter cloth and cause a very high resistance to flow.
- (c) The orientation of the particle in the initial layers may appreciably influence the structure of the whole filter cake.

Filter cakes may be divided into two classes—incompressible cakes and compressible cakes. In the case of an incompressible cake, the resistance to flow of a given volume of cake is not appreciably affected either by the pressure difference across the cake or by the rate of deposition of material. On the other hand, with a compressible cake, increase of the pressure difference or of the rate



of flow causes the formation of a denser cake with a higher resistance. For incompressible cakes  $e$  in equation (1) may be taken as constant and the quantity is then a property of the particles forming the cake and should be constant for a given material.

$$\text{Thus: } \frac{1}{A} \frac{dV}{dt} = \frac{-\Delta P}{r \mu l} \quad \dots\dots\dots 2$$

$$e^3/[5(1-e)^2 S^2]$$

$$\text{Where: } r = \frac{5(1-e)^2 S^2}{e^3} \quad \dots\dots\dots 3$$

It may be noted that, when there is a hydrostatic pressure component such as with a horizontal filter surface, this should be included in the calculation of  $-\Delta P$ . Equation (2) is the basic filtration equation and  $r$  is termed the specific resistance which is seen to depend on  $e$  and  $S$ . For incompressible cakes,  $r$  is taken as constant, although it depends on rate of deposition, the nature of the particles, and on the forces between the particles.

#### 4.4.1 Relation between thickness of cake and volume of filtrate.

In equation 7.2, the variables  $l$  and  $V$  are connected, and the relation between them may be obtained by making a material balance between the solids in both the slurry and the cake as follows.

$$\begin{aligned} \text{Volume of solids in filter cake} &= (\text{volume of cake}) - (\text{volume of pores}) \\ &= Al - Ale = (1-e) Al \dots\dots\dots 4a \end{aligned}$$

$$\text{Mass of solids in filter cake} = (1-e) Al \rho_s \dots\dots\dots 4b$$

where  $\rho_s$  is the density of the solids

$$\text{Mass of liquid retained in the filter cake} = eAl\rho \dots\dots\dots 5$$

Where  $\rho$  is the density of the filtrate.

If  $J$  is the mass fraction of solids in the original suspension then:

$$(1-e)lA\rho_s = \frac{(V + eAl)\rho J}{1-J} \quad \dots\dots\dots 6$$

$$(1-J)(1-e)Al\rho_s = JV\rho + AeJl\rho \quad \dots\dots\dots 7$$

$$\text{So that: } l = \frac{JV\rho}{A\{(1-J)(1-e)\rho_s - Je\rho\}} \quad \dots\dots\dots 8$$

$$\text{and: } V = \frac{\{\rho_s(1-e)(1-J) - e\rho J\}Al}{\rho J} \quad \dots\dots\dots 9$$

If  $v$  is the volume of cake deposited by unit volume of filtrate then:

$$v = \frac{lA}{V} \quad \text{or} \quad l = \frac{vV}{A} \quad \dots\dots\dots 10$$

and from equation(9)

$$v = \frac{J\rho}{(1-J)(1-e)\rho_s - J e \rho} \quad \dots\dots\dots 11$$

Substituting for  $l$  in equation(2)

$$\frac{1}{A} \frac{dV}{dt} = \frac{(-\Delta P)}{r\mu} \frac{A}{vV}$$

$$\text{or} \quad \frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V} \quad \dots\dots\dots 12$$

Equation (12) may be regarded as the basic relation between  $-\Delta P$ ,  $V$ , and  $t$ . Two important types of operation are: (i) where the pressure difference is maintained constant and (ii) where the rate of filtration is maintained constant.

*For a filtration at constant rate:-*

$$\frac{dV}{dt} = \frac{V}{t} = \text{constant}$$

$$\text{So} \quad \text{that:} \quad \frac{V}{t} = \frac{A^2(-\Delta P)}{r\mu V v} \quad \dots\dots\dots 13$$

$$\text{or} \quad \frac{t}{V} = \frac{r\mu v}{A^2(-\Delta P)} V \quad \dots\dots\dots 14$$

and  $-\Delta P$  is directly proportional to  $V$ .

*For a filtration at constant pressure difference*

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V}$$

$$\frac{V^2}{2} = \frac{A^2(-\Delta P)t}{r\mu v} \quad \dots\dots\dots 15$$

$$\text{or:} \quad \frac{t}{V} = \frac{r\mu v}{2A^2(-\Delta P)} V \quad \dots\dots\dots 16$$

Thus for a constant pressure filtration, there is a linear relation between  $V^2$  and  $t$  or between  $t/V$  and  $V$ .

Filtration at constant pressure is more frequently adopted in practice, although the pressure difference is normally gradually built up to its ultimate value.

If this takes a time  $t_1$  during which a volume  $V_1$  of filtrate passes, then integration of equation (16) gives:

$$\frac{1}{2}(V^2 - V_1^2) = \frac{A^2(-\Delta P)}{r\mu v}(t - t_1) \quad \dots\dots\dots 17$$

$$\text{or: } \frac{t - t_1}{V - V_1} = \frac{r\mu v}{2A^2(-\Delta P)}(V - V_1) + \frac{r\mu v V_1}{A^2(-\Delta P)} \dots\dots\dots 18$$

Thus, there where is a linear relation between  $V^2$  and  $t$ , and between  $(t - t_1)/(V - V_1)$  and  $(V - V_1)$ ,

Where :

$(t - t_1)$  represents the time of the constant pressure filtration  
 $(V - V_1)$  the corresponding volume of filtrate obtained.

#### 4.4.2. Flow of liquid through the cloth.

Experimental work on the flow of the liquid under streamline conditions has shown that the flow rate is directly proportional to the pressure difference. It is the resistance of the cloth plus initial layers of deposited particles that is important since the latter, not only form the true medium, but also tend to block the pores of the cloth thus increasing. It is therefore usual to combine the resistance of the cloth with that of the first few layers of particles and suppose that this corresponds to a thickness  $L$  of cake as deposited at a later stage. The resistance to flow through the cake and cloth combined is now considered.

#### 4.4.3 Flow of filtrate through the cloth and cake combined.

If the filter cloth and the initial layers of cake are together equivalent to a thickness  $L$  of cake as deposited at a later stage in the process, and if  $-\Delta P$  is the pressure drop across the cake and cloth combined, then:

$$\frac{1}{A} \frac{dV}{dt} = \frac{(-\Delta P)}{r\mu(L + L_e)} \dots\dots\dots 19$$

which may be compared with equation (2).

$$\text{Thus: } \frac{dV}{dt} = \frac{A(-\Delta P)}{r\mu\left(\frac{Vv}{A} + \frac{V_e v}{A}\right)} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)} \dots\dots\dots 20$$

For the period of *constant rate filtration*: equation(20) may be integrated between the limits  $t = 0, V = 0$  and  $t = t_1, V = V_1$

$$\frac{V_1}{t_1} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)}$$

or:

$$\frac{t_1}{V_1} = \frac{r\mu v}{A^2(-\Delta P)} V_1 + \frac{r\mu v}{A^2(-\Delta P)} V_e$$

or: 
$$V_1^2 + V_e V_1 = \frac{A^2(-\Delta P)}{r\mu v} t_1 \dots\dots\dots 21$$

For a subsequent *constant pressure filtration*:

Equation(20) may be integrated between the limits  $t = 0, V = 0$  and  $t = t_1, V = V_1$

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)} \dots\dots\dots 20$$

$$\int_0^{V_1} (V + V_e) dV = \frac{A^2(-\Delta P)}{r\mu v} \int_0^{t_1} dt$$

$$\frac{V_1^2}{2} + V_e V_1 = \frac{A^2(-\Delta P)}{r\mu v} t \dots\dots\dots 22a$$

$$t = \frac{r\mu v}{A^2(-\Delta P)} \left( \frac{V_1^2}{2} + V_e V_1 \right) \dots\dots\dots 22b$$

For a subsequent *constant pressure filtration*: stating at  $V_1$  the equation (20) integrated between  $t = t_1, V = V_1$  and  $t = t, V = V$ :

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)}$$

$$\int_{V_1}^V (V + V_e) dV = \frac{A^2(-\Delta P)}{r\mu v} \int_{t_1}^t dt$$

$$\frac{1}{2}(V^2 - V_1^2) + V_e (V - V_1) = \frac{A^2(-\Delta P)}{r\mu v} (t - t_1) \dots\dots\dots 7.23$$

or:

$$(V - V_1 + 2V_1)(V - V_1) + 2V_e (V - V_1) = \frac{2A^2(-\Delta P)}{r\mu v} (t - t_1)$$

or: 
$$\frac{t - t_1}{V - V_1} = \frac{r\mu v}{2A^2(-\Delta P)} (V - V_1) + \frac{r\mu v}{A^2(-\Delta P)} (V_1 + V_e) \dots\dots\dots 7.24$$

Thus there is a linear relation between  $(t - t_1)/(V - V_1)$  and  $V - V_1$ , and the slope is proportional to the specific resistance, as in the case of the flow of the filtrate through the filter cake alone given by equation 7.18, although the line does not now go through the origin.

## 4.5. FILTRATION PRACTICE.

1. The filter medium
2. Blocking filtration
3. Effect of particle sedimentation on filtration
4. Delayed cake filtration
5. Cross-flow filtration
6. Preliminary treatment of slurries before filtration
7. Washing of the filter cake

### Washing of the filter cake

When the wash liquid is miscible with the filtrate and has similar physical properties, the rate of washing at the same pressure difference will be about the same as the final rate of filtration. If the viscosity of the wash liquid is less, a somewhat greater rate will be obtained. Channeling sometimes occurs, however, with the result that much of the cake is incompletely washed and the fluid passes preferentially through the channels, which are gradually enlarged by its continued passage. This does not occur during filtration because channels are self-sealing by virtue of deposition of solids from the slurry. Channeling is most marked with compressible filter cakes and can be minimized by using a smaller pressure difference for washing than for filtration.

Washing may be regarded as taking place in two stages. First, filtrate is displaced from the filter cake by wash liquid during the period of *displacement washing* and in this way

up to 90 per cent of the filtrate may be removed. During the second stage, *diffusional washing*, solvent diffuses into the wash liquid from the less accessible voids and the following relation applies:

$$\left( \frac{\text{volume of wash liquid passed}}{\text{cake thickness}} \right) = (\text{constant}) \times \log \left( \frac{\text{initial concentration of solute}}{\text{concentration at particular time}} \right)$$

Although an immiscible liquid is seldom used for washing, air is often used to effect partial drying of the filter cake.

### In case of filter press:

Two methods of washing may be employed, “simple” washing and “through” or “thorough” washing.

**Simple washing**, the wash liquid is fed in through the same channel as the slurry although, as its velocity near the point of entry is high, erosion of the cake takes place. The channels which are thus formed gradually enlarge and uneven washing is usually obtained. Simple washing may be used only when the frame is not completely full.

In **thorough washing**, the wash liquid is introduced through a separate channel behind the filter cloth on alternate plates, known as washing plates and flows through the whole thickness of the cake, first in the opposite direction and then in the same direction as the filtrate. The area during washing is one-half of that

during filtration and, in addition, the wash liquid has to flow through twice the thickness, so that the rate of washing should therefore be about **one-quarter** of the final rate of filtration.

$$\left. \frac{dV}{dt} \right|_w = \frac{1}{4} \times \left. \frac{dV}{dt} \right|_f$$

**In case of rotary drum filter:**

For continuous filtration the resistance to the filter medium is negligible compared with the resistance to the filter cake so: the rate of washing in rotary drum filter is **equal** to the rate of filtration

$$\left. \frac{dV}{dt} \right|_w = \left. \frac{dV}{dt} \right|_f$$

**Total time of filtration**

**For batch filtration:**

Total time = time of filtration + time of washing + time of cleaning + time of resample + time of dismantle.

**For continuous filtration:** such as rotary drum, and pressure drop is held constant for the filtration, the filtration time  $t$  is less than total cycle time  $t_c$

$$t = f t_c$$

where  $f$  in rotary drum is the fraction submergence of the drum surface in the slurry

## 4.6 The filter press.

The filter press is one of two main types, the *plate and frame press* and the *recessed plate or chamber press*.

***The plate and frame press***

The optimum thickness of cake to be formed in a filter press depends on the resistance offered by the filter cake and on the time taken to dismantle and refit the press. Although the production of a thin filter cake results in a high average rate of filtration, it is necessary to dismantle the press more often and a greater time is therefore spent on this operation. For a filtration carried out entirely at constant pressure, a rearrangement equation 7.22a gives:

$$\frac{t}{V} = \frac{r\mu v}{2A^2(-\Delta P)} V + \frac{r\mu L}{A(-\Delta P)} \dots\dots\dots 25a$$

$$= B_1 V + B_2 \dots\dots\dots 25b$$

where  $B_1$  and  $B_2$  are constants.

Thus the time of filtration  $t$  is given by:

$$t = B_1 V^2 + B_2 V \quad \dots\dots\dots 26$$

The time of dismantling and assembling the press, say  $t'$  is substantially independent of the thickness of cake produced. The total time of a cycle in which a volume  $V$  of filtrate is collected is then  $(t + t')$  and the overall rate of filtration is given by:

$$W = \frac{V}{B_1 V^2 + B_2 V + t'} \quad \dots\dots\dots 27$$

$W$  is a maximum when  $dW/dV = 0$ .

Differentiating  $W$  with respect to  $V$  and equating to zero:

$$B_1 V^2 + B_2 V + t' - V(2B_1 V + B_2) = 0$$

$$t' = B_1 V^2 \quad \dots\dots\dots 28$$

$$\text{or} \quad V = \sqrt{\left(\frac{t'}{B_1}\right)} \quad \dots\dots\dots 29$$

If the resistance of the filter medium is neglected,  $t = B_1 V^2$  and the time during which filtration is carried out is exactly equal to the time the press is out of service. In practice, in order to obtain the maximum overall rate of filtration, the filtration time must always be somewhat greater in order to allow for the resistance of the cloth, represented by the term  $B_2 V$ . In general, the lower the specific resistance of the cake, the greater will be the economic thickness of the frame.

## 4.6 Rotary – Drum filter.

This type of filter is used for continuous operation ,where in this type the feed ,filtrate , and the cake move at steady ,continuous rate. In a rotary drum the pressure drop is held constant for the filtration. The cake formation involves a continual change in conditions. In continuous filtration the resistance of the filter medium is generally negligible compared with the cake resistance. So the rate of filtration is :-

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V}$$

$$\frac{V^2}{2} = \frac{A^2(-\Delta P)t}{r\mu v} \quad \dots\dots\dots 15$$

$$\frac{t}{V} = \frac{r\mu v}{2A^2(-\Delta P)} V \quad \dots\dots\dots 16$$

Where  $t$  is the time required for the formation of the cake ,while in rotary drum filter, the filter time  $t$  is less than the total cycle time  $t_c$  as:-

$$t = f t_c$$

where  $f$  in rotary drum is the fraction submergence of the drum surface in the slurry. Therefore the rate of filtration is :-

$$\frac{V^2}{2} = \frac{A^2 (-\Delta P) f t_c}{r \mu v} \dots\dots\dots 30$$

Rearrange equation (30) :-

$$\frac{V}{t_c} = A \left[ \frac{2 f (-\Delta P)}{t_c r \mu v} \right]^{1/2} \dots\dots\dots 31$$

When short times are used in continuous filtration and/or the filter medium resistance is relatively large , the filter resistance term must be included :-

$$\frac{dV}{dt} = \frac{A^2 (-\Delta P)}{r \mu v (V + V_e)} \dots\dots\dots 20$$

$$t = \frac{r \mu v}{A^2 (-\Delta P)} \left( \frac{V_1^2}{2} + V V_e \right) \dots\dots\dots 22b$$

and equation (22b)become:-

$$\frac{V}{t_c} = A \left( \frac{-V_e/t_c + \left[ V_e^2/t_c^2 + 2rv(-\Delta P)f/\mu t_c \right]^{1/2}}{rv} \right) \dots\dots\dots 23$$



**Example 1**

A slurry is filtered in a plate and frame press containing 12 frames, each 0.3 m square and 25 mm thick. During the first 180 s the pressure difference for filtration is slowly raised to the final value of  $400 \text{ kN/m}^2$  and, during this period, the rate of filtration is maintained constant. After the initial period, filtration is carried out at constant pressure and the cakes are completely formed in a further 900 s. The cakes are then washed with a pressure difference of  $275 \text{ kN/m}^2$  for 600 s using *thorough washing* (See the plate and frame press in Section 7.4.4). What is the volume of filtrate collected per cycle and how much wash water is used? A sample of the slurry had previously been tested with a leaf filter of  $0.05 \text{ m}^2$  filtering surface using a vacuum giving a pressure difference of  $71.3 \text{ kN/m}^2$ . The volume of filtrate collected in the first 300 s, was  $250 \text{ cm}^3$  and, after a further 300 s, an additional  $150 \text{ cm}^3$  was collected. It may be assumed that the cake is incompressible and that the cloth resistance is the same in the leaf as in the filter press.

**Example 2**

Sludge is filtered in a plate and frame press fitted with 25 mm frames. For the first 600 s the slurry pump runs at maximum capacity. During this period the pressure rises to  $415 \text{ kN/m}^2$  and 25 per cent of the total filtrate is obtained. The filtration takes a further 3600 s to complete at constant pressure and 900 s is required for emptying and resetting the press. It is found that if the cloths are pre-coated with filter aid to a depth of 1.6 mm, the cloth resistance is reduced to 25 per cent of its former value. What will be the increase in the overall throughput of the press if the pre-coat can be applied in 180 s?

# Transport Phenomena

## CHAPTER 5

### *Liquid–Liquid Extraction*

#### **5.1 Introduction.**

The separation of the components of a liquid mixture by treatment with a solvent in which one or more of the desired components is preferentially soluble is known as liquid–liquid extraction—an operation which is used, for example, the separation of hydrocarbons in the petroleum industry and in chemical industries, such as separation of aromatics from kerosene-based fuel oils to improve their burning qualities and the separation of aromatics from paraffin and naphthenic compounds to improve the temperature-viscosity characteristics of lubricating oils. It may also be used to obtain relatively pure compounds such as benzene, toluene, and xylene from catalytically produced reformats in the oil industry and in the metallurgical and biotechnology industries.

In this operation, it is essential that the liquid-mixture feed and solvent are at least partially if not completely immiscible and, in essence, three stages are involved:

- (a) Bringing the feed mixture and the solvent into intimate contact.
- (b) Separation of the resulting two phases.
- (c) Removal and recovery of the solvent from each phase.

It is possible to combine stages (a) and (b) into a single piece of equipment such as a column which is then operated continuously. Such an operation is known as differential contacting. Liquid–liquid extraction is also carried out in stagewise equipment, the prime example being a mixer–settler unit in which the main features are the mixing of the two liquid phases by agitation, followed by settling in a separate vessel by gravity. The extraction operation may be either a physical operation, or a chemical operation.

Extraction is in many ways complementary to distillation and is preferable in the following cases:

- (a) Where distillation would require excessive amounts of heat, such as, for example, when the relative volatility is near unity.
- (b) When the formation of azeotropes limits the degree of separation obtainable in distillation.
- (c) When heating must be avoided.
- (d) When the components to be separated are quite different in nature.

(e) When one of the components is present at such low concentration that distillation becomes not possible and economic.

\*For high – molecular weight fatty acids can be separated from vegetable oils by extraction with liquid propane or by high vacuum distillation ,which is more expensive.

## 5.2 Choice of Solvent.

1. **Selectivity** :-the effectiveness of solvent B for separating a solution of A and C (desired components) is measured by comparing the ratio of C to A in the B-rich phase to that in A-rich phase at equilibrium. For all useful extraction operation ,the selectivity must exceed unity , the more so that the better .If the selectivity is unity , no separation is possible.
2. **Recoverability**.
3. **Density**:-the larger difference in densities of the saturated liquid phases I necessary.
4. **Interfacial tension** :-should be high , due to more readily coalescence of emulsions will occur but the more difficult the dispersion of one liquid in the other will be.
5. **Chemical reactivity** :- should be stable chemically and inert toward the other components of system and material of construction.
6. **Viscosity, Vapor Pressure , Freezing Point** :-should be low for ease in handling and storage.
7. **The solvent should be antitoxic, nonflammable and low cost.**

## 5.3 Extraction Processes.

The three steps outlined in Section above 5.1, necessary in all liquid-liquid extraction operations, may be carried out either as a batch or as a continuous process. In the single-stage batch process illustrated in Figure 5.1, the solvent (B) and solution(A and C) are mixed together and two liquid layers formed which insoluble, then allowed to separate into the two phases—the *extract* E containing the required solute in the added solvent(rich-B) and the *raffinate* R, the weaker solution with some associated solvent(rich-A). With this simple arrangement mixing and separation occur in the same vessel.

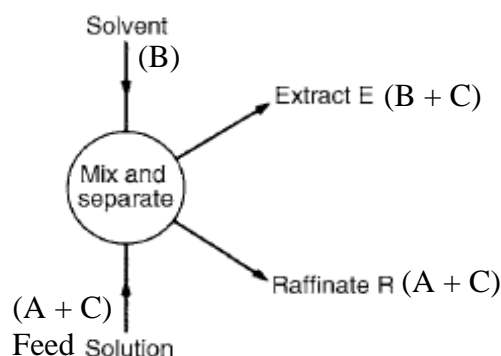


Figure 5.1. Single-stage batch extraction.

A continuous multistage such as two-stage operation is shown in Figure 5.2, where the mixers and separators are shown as separate vessels. There are three main forms of equipment. First there is the mixer-settler as shown in Figure 5.1, secondly, there is the column type of design with trays or packing as in distillation and, thirdly, there are a variety of units incorporating rotating devices. In all cases, the extraction units are followed by distillation or a similar operation in order to recover the solvent and the solute. Some indication of the form of these alternative arrangements may be seen by considering two of the processes referred to in Section 5.1.

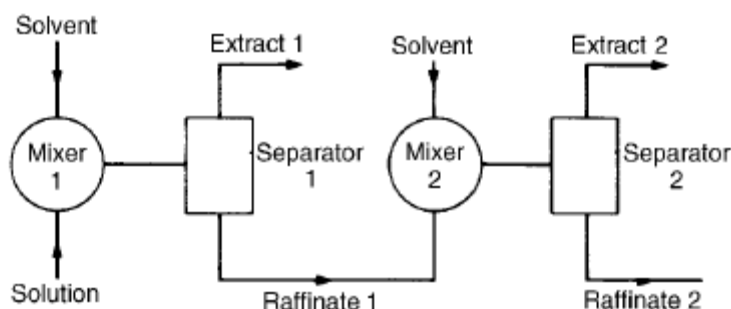


Figure 5.2. Multiple-contact system with fresh solvent.

## 5.4 Equilibrium Data.

Extraction involve the use of systems composed of at least three substances and although for the most part the insoluble phase are chemically very differently, generally all three components appear at least to some extent in both phases. The following notation scheme will be used to describe the concentrations and amounts of these ternary mixture, for purposes of discussing both material balance and equilibrium.

- A and B are pure, substantially insoluble liquids, and C is the distributed solute. Mixtures to be separated by extraction are composed of A and C, and B is the extracting solvent.
- Quantities are measured by mass for batch operations, mass/time for continuous operation.

$E$  = mass/ time, extract.

$R$  = mass/ time, raffinate.

$B$  = mass/ time, solvent.

$E'$  = mass B-free/ time.

$x$  = weight fraction C in the solvent-lean (rich-A).

$y$  = weight fraction C in the solvent-rich (rich-B).

$$x' = \frac{x}{1-x} = \frac{\text{mass } C}{\text{mass non-C in raffinate}}$$

$$y' = \frac{y}{1-y} = \frac{\text{mass } C}{\text{mass non-C in extract}}$$

$$X = \text{weight fraction C in raffinate on a B-free basis} = \frac{\text{mass C}}{\text{mass A} + \text{mass C}}$$

$$Y = \text{weight fraction C in extract on a B-free basis} = \frac{\text{mass C}}{\text{mass A} + \text{mass C}}$$

$$N = \text{weight fraction B on a B-free basis} = \frac{\text{mass B}}{\text{mass A} + \text{mass C}}$$

The addition of a new solvent to a binary mixture of a solute in a solvent may lead to the formation of several types of mixture:

- A homogeneous solution may be formed and the selected solvent is then unsuitable.
- The solvent may be completely immiscible with the initial solvent.
- The solvent may be partially miscible with the original solvent resulting in the formation of one pair of partially miscible liquids.
- The new solvent may lead to the formation of two or three partially miscible liquids.

With conditions of type (b), the equilibrium relation is conveniently shown by a plot of the concentration of solute in one phase against the concentration in the second phase. Conditions given by (c) and (d) are usually represented by triangular diagrams.

### Equilibrium Data on Triangular Coordinate.

#### *One pair of partially miscible liquids.*

The system, acetone (C)–Water (A)–methyl isobutyl ketone (B), as shown in Figure 5.3a, is of type (c).

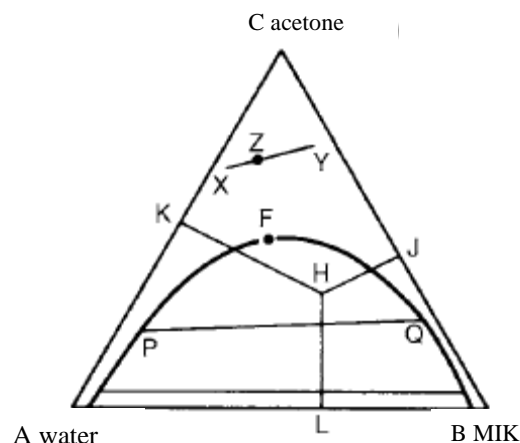


Figure 5.3a

Here the solute **C** is completely miscible with the two solvents **A** and **B**, although the two solvents are only partially miscible with each other. Each point inside the triangle represents the composition of a possible three-component mixture, a

mixture indicated by point H consists of the three components **A**, **B** and **C** in the ratio of the perpendiculars HL, HJ, HK. A point at one of the three corners represents a pure component, while the normal distance between an interior point and the side opposite the corner for pure A represents the mole fraction or mass fraction of component A in the mixture. The distance HL represents the mass fraction  $X_C$  of C in the mixture at H, HJ represents  $X_B$ , HK represents  $X_A$ .

Where  $X_C + X_B + X_A = 1$

The area under the curved line NPFQM, the binodal solubility curve, represents a two-phase region which will split up into two layers in equilibrium with each other. These layers have compositions represented by points P and Q, and PQ is known as a “tie line”. Such lines, two of which are shown in the diagram, connect the compositions of two phases in equilibrium with each other, and these compositions must be found by practical measurement. There is one point on the binodal curve at F which represents a single phase that does not split into two phases. F is known as a *plait* point, and this must also be found by experimental measurement. The plait point is fixed if either the temperature or the pressure is fixed.

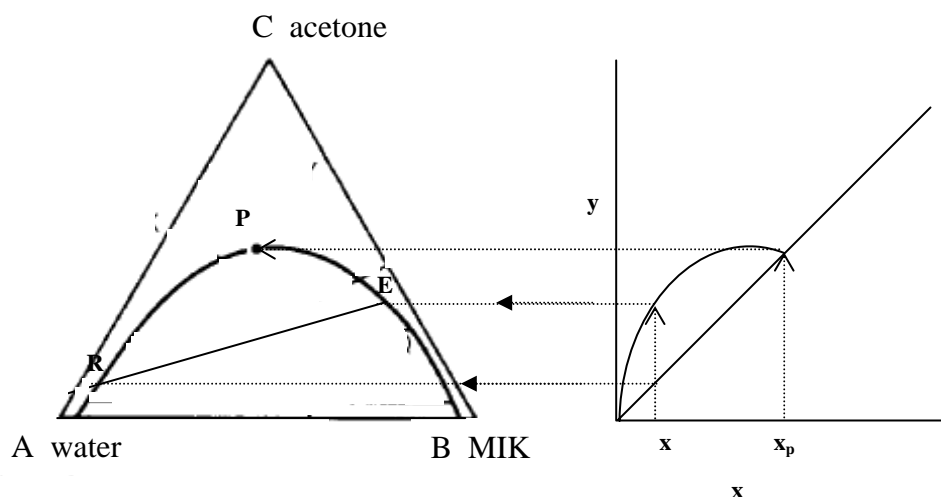
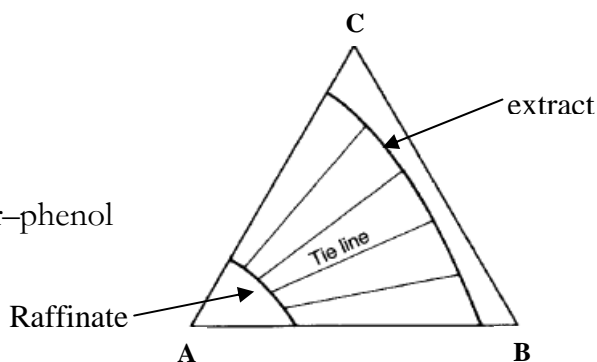


Figure 5.3b

### *Two or three partially miscible liquids.*

In Figure 5.4 the two-phase regions merge on varying the temperature. Aniline (**A**), water (**B**), and phenol (**C**) represent a system of the latter type. Under the conditions shown in Figures 5.4, **B** and **C** are miscible in all proportions, although **B** and **A**, and **A** and **C** are only partially miscible. Figure 5.4 shows equilibrium relationship for the aniline–water–phenol at high temperature.

Figure 5.4  
relationship for the aniline–water–phenol



## Equilibrium Data on Rectangular Coordinate.

In some system equilateral triangular coordinates have some disadvantage, therefore special cases a more useful method for plotting the three component data using rectangular coordinate as in acetic acid(A)-water(B)-isopropyl ether(C) as shown in Figure 5.5.

The solvent pair B and C partially miscible.

The concentration of component C is plotted

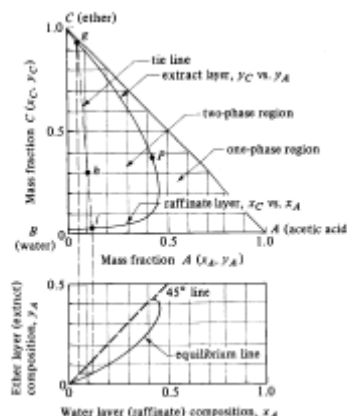
in a vertical axis and A in a horizontal axis,

while the concentration of B can be obtained as:-

$$X_B = 1 - X_A - X_C$$

$$Y_B = 1 - Y_A - Y_C$$

The line gi is called the tie line, I represent the water Rich layer(raffinate)and j is the solvent rich layer (isopropyl ether).The tie line also drawn by distribution Diagram y vs. x plotted below the phase diagram.



- ❖ The system another rectangular system can be plotted on a B-free basis using  $x$ ,  $y$  diagram. The equilibrium data (solubility data) in extraction, these coordinate are plotted on a B-free basis (solvent free basis). These coordinate are represented by  $x$ ,  $y$ ,  $N$ , where  $x$  is A rich,  $y$  is B rich and  $N$  is the B concentration on a B-free basis as shown on Figure 5.5.

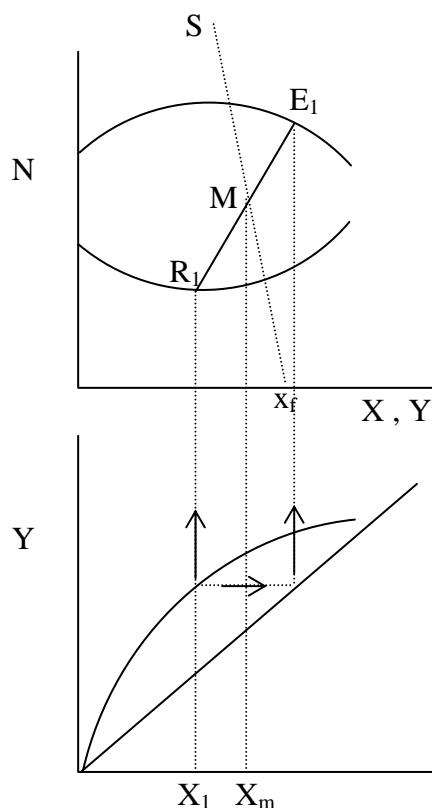


Figure 5.5

## 5.5 Calculation of the Number of Theoretical Stages.

### 5.5.1. Co-current contact with partially miscible solvents (Single stage).

Extraction equipment of stage-wise type can be carried out according to

- a- The nature of the system.
- b- The extent of separation.

It is to be understood that the effluent extract and raffinate solutions are in equilibrium with each other. Each stage must include facilities for contacting the insoluble liquids and separating product streams. A combination of a mixer and a settler may therefore constitute a stage. This may be batch or continuous operation.

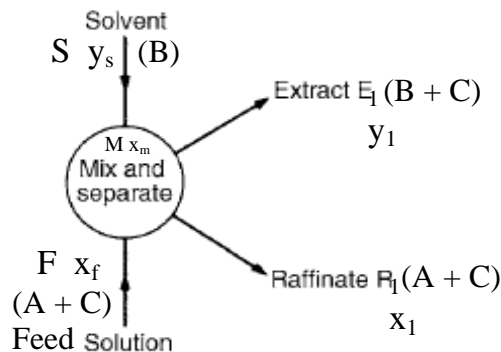


Figure 5.6

**F** - mass of feed (batch) or mass of feed/time(continuous) .It contain substances(A+C) at  $x_f$  weight fraction C.

**S** – mass of solvent (batch) or mass of solvent/time (continuous) . The main constituent of this stream is B , containing  $y_s$  weight fraction.

**R<sub>1</sub>** – mass of raffinate (batch) or mass of raffinate/time (continuous) , $x_1$  is the mass fraction of C in raffinate.

**E<sub>1</sub>** – mass of extract (batch) or mass of extract /time (continuous) , $y_1$  is the mass fraction of C in extract.

- ❖ Adding **S** and **F** produces in the extraction stage a mixture **M<sub>1</sub>** which on settling forms the equilibrium phases **E<sub>1</sub>** and **R<sub>1</sub>** joined by the tie line through **M<sub>1</sub>**.
- ❖ If the solvent is pure B( $y_s=0$ ), it will be plotted at the B apex, but sometimes it has been recovered from a previous extraction and therefore contain a little A and C as well ( $y_s \neq 0$ ).



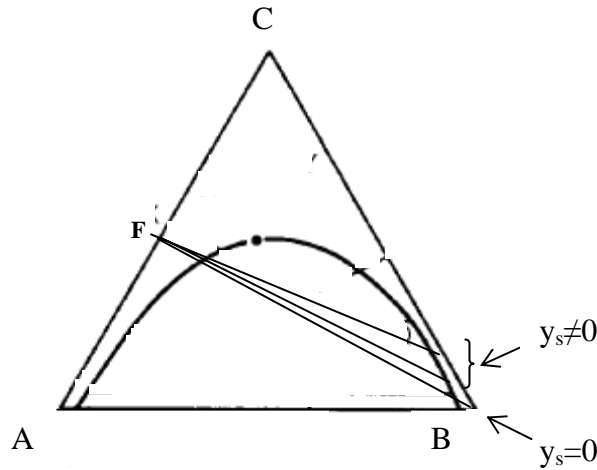


Figure 5.7

❖ **Material Balance.**

- **Total material balance**

$$\mathbf{F} + \mathbf{S} = \mathbf{M}_1 = \mathbf{E}_1 + \mathbf{R}_1 \dots \dots \dots (1)$$

$\mathbf{M}_1$  is located on the FS line .

- **C-balance.**

$$\mathbf{F} x_f + \mathbf{S} y_s = \mathbf{M}_1 x_{m1} = \mathbf{E}_1 y_1 + \mathbf{R}_1 x_1 \dots \dots \dots (2)$$

$x_{m1}$  is the mass fraction of C in the mixture.

$$x_{m1} = \frac{F x_f + S y_s}{F + S} \dots \dots \dots (3)$$

The amount of solvent to provide a location for  $\mathbf{M}_1$  on the line  $\mathbf{FS}$  can be complete

$$\mathbf{F} x_f + \mathbf{S} y_s = \mathbf{M}_1 x_{m1} \dots \dots \dots (4)$$

$\mathbf{F} + \mathbf{S} = \mathbf{M}_1$  .....sub in (4); then:-

$$\frac{S}{F} = \frac{x_f - x_{m1}}{x_{m1} - y_s} \dots \dots \dots (5a)$$

For pure solvent  $\mathbf{S}=\mathbf{B}$  ,  $y_s=0$ ...then equation(5a) become :-

$$\frac{S}{F} = \frac{x_f - x_{m1}}{x_{m1}} \dots \dots \dots (5b)$$

The quantities of extract and raffinate :-

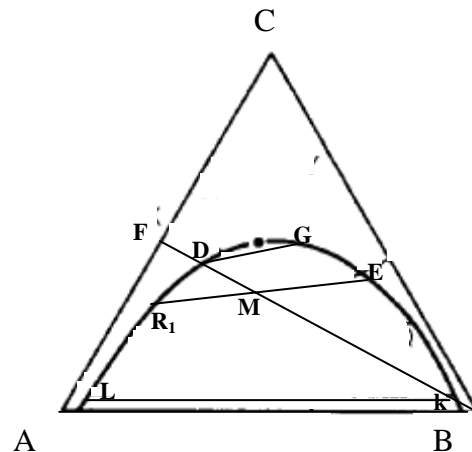
$$\mathbf{E}_1 + \mathbf{R}_1 = \mathbf{M}_1 \dots \dots \dots (6a)$$

$$\mathbf{E}_1 y_1 + \mathbf{R}_1 x_1 = \mathbf{M}_1 x_{m1} \dots \dots \dots (6b)$$

From equations 6a and 6b  $\mathbf{E}_1$  and  $\mathbf{R}_1$  can be estimated as:-

$$E_1 = \frac{M_1(x_{m1} - x_1)}{y_1 - x_1} \dots \dots \dots (7a) , \quad R_1 = \frac{M_1(x_{m1} - y_1)}{x_1 - y_1} \dots \dots \dots (7b)$$

Since two insoluble phases must form for an extraction operation , point **M<sub>1</sub>** must lie within the heterogeneous liquid area , as shown:-



The minimum amount of solvent is found by locating **M<sub>1</sub>** at **D** , which would then provide an infinitesimal amount of extract at **G** the maximum amount of solvent is found by locating **M<sub>1</sub>** at **k** , which provides an infinitesimal amount of raffinate at **L**. Point L represents the raffinate with lowest possible C concentration , and if a lower value were required , the record solvent S should to have lower concentration.

- All the computation can be made on a B-free basis(see figure(5.5)) as follows:-  
Locate **F** and s point ,join **FS** line ,if the solvent **S** is pure **B** ,its **N** value is infinite and the line **FS** is then vertical. The product **E<sub>1</sub>** and **R<sub>1</sub>** lie on a line through **M<sub>1</sub>** represents the entire mixture in the extractor.

- Material Balance(B-Free basis).

$$F' + S' = M'_1 = E'_1 + R'_1 \dots \dots \dots (8)$$

The feed is normally free of **B** and **F = F'**

- **C-Balance.**

$$F' X_f + S' Y_s = M'_1 X_{m1} = E'_1 Y_1 + R'_1 X_1 \dots (9)$$

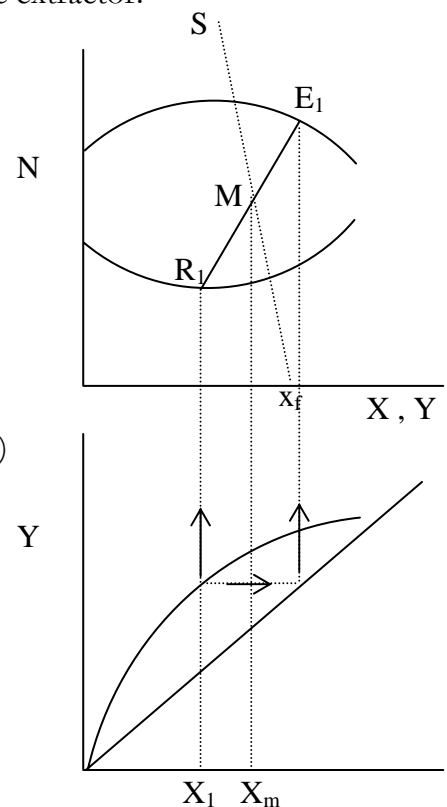
- **B-Balance.**

$$F' N_f + S' N_s = M'_1 N_{m1} = E'_1 N_{E1} + R'_1 N_{R1} \dots (10)$$

Since the feed no contain **B** , **N<sub>f</sub> = 0**.

From equations 8,9,and10 ;**E'<sub>1</sub>** and **R'<sub>1</sub>** can be estimated as :-

$$E'_1 = \frac{M'_1 (X_{m1} - X_1)}{Y_1 - X_1} \dots \dots \dots (11a)$$



$$R'_1 = \frac{M'_1(X_{m_1} - Y_1)}{X_1 - Y_1} \dots\dots\dots(11b)$$

$$\frac{R'_1}{E'_1} = \frac{Y_1 - X_{m_1}}{X_{m_1} - X_1} \dots\dots\dots(12)$$

The total weight of the saturated extract and raffinate are :-

$$E_1 = E'_1(1 + N_{E1}) \dots\dots\dots(13a)$$

$$R_1 = R'_1(1 + N_{R1}) \dots\dots\dots(13b)$$

### 5.5.2. Multistage Cross-current contact with partially miscible solvents.

This is an extension of single-stage extraction wherein the raffinate is successively contacted with fresh solvent, and may be done continuously or in batch.

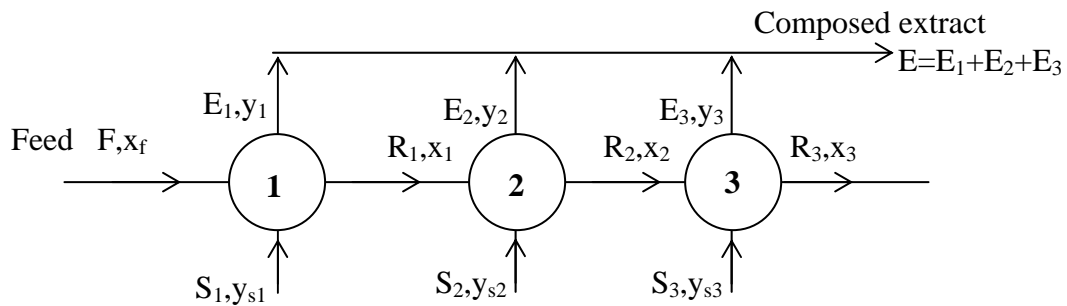


Figure5.8

**Material balance for each stage:-**

$$F + S_1 = M_1 = E_1 + R_1 \dots\dots\dots(14)$$

$$R_1 + S_2 = M_2 = E_2 + R_2 \dots\dots\dots(15)$$

$$R_2 + S_3 = M_3 = E_3 + R_3 \dots\dots\dots(16)$$

$$\text{Composed extract } E = E_1 + E_2 + E_3 \dots\dots\dots(17)$$

**For n stage :-**

Total material balance.

$$R_{n-1} + S_n = M_n = E_n + R_n \dots\dots\dots(18)$$

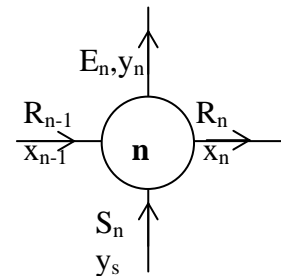
C – balance.

$$R_{n-1} x_{n-1} + S_n y_s = M_n x_{mn} = E_n y_n + R_n x_n \dots\dots\dots(19)$$

For solvent free coordinates:-

A+C balance

$$R'_{n-1} + S'_n = M'_n = E'_n + R'_n \dots\dots\dots*$$



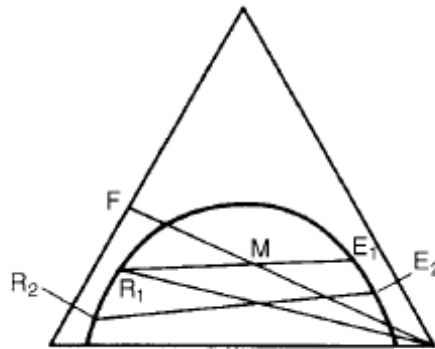
C – balance.

$$R'_{n-1} X_{n-1} + S'_n Y_s = M'_n X_{mn} = E'_n Y_n + R'_n X_n \dots\dots\dots **$$

B – balance.

$$R'_{n-1} N_{n-1} + S'_n N_s = M'_n N_{mn} = E'_n N_{En} + R'_n N_{Rn} \dots\dots (20)$$

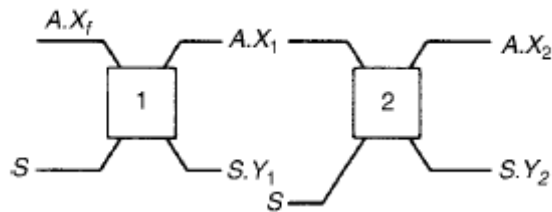
Unequal amount of solvent can be used in various stages , and even different temperatures in which case each stage must be computed with the help of a phase diagram at the appropriate concentration , the greater the number of stages the less total solvent will be used.



### 5.5.3. Co-current contact with immiscible solvents.

When the extraction solvent and the feed solution are insoluble and remain so at all concentrations of the distributed solute occurring in the operation .For this purpose, the equilibrium concentrations are plotted as  $x' (x/(1-x))$  vs.  $y'(y/(1-y))$  . Since the liquids A and B are in soluble , there are A kg of this substance in all raffinate. Similarly the extract from each stage contain all the solvent B fed to that stage.

Figure 5.9. For completely insoluble liquid

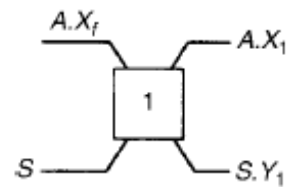


A solute C-balance about any stage n :-

$$A x'_{n-1} + B_n y'_s = B_n y'_n + A x'_n \dots\dots\dots (21)$$

$$A (x'_{n-1} - x'_n) = B_n (y'_n - y'_s)$$

$$-\frac{A}{B_n} = \frac{y'_s - y'_n}{x'_{n-1} - x'_n} \dots\dots\dots (22)$$



This is the operating line equation for stage n of slope  $-A/B_n$  , passing through points  $(x'_{n-1} , y'_s)$  and  $(y'_n , x'_n)$ . The concentration for a three stage extraction is

shown in figure 5.10, where for each stage a line is drawn of slope appropriate to that stage. Each operation line intersects the equilibrium curve at the raffinate and extract compositions.

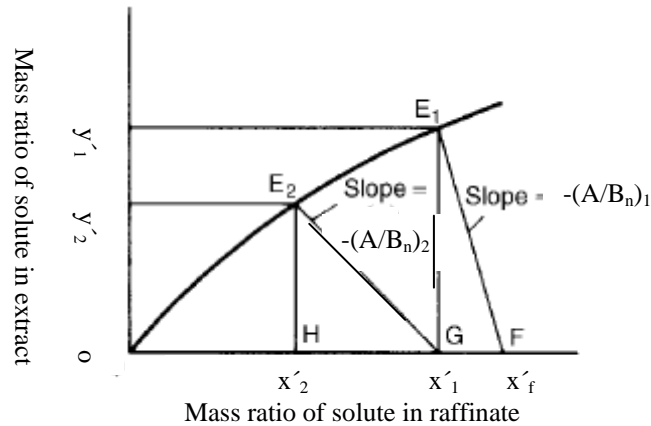


Figure 5.10

This process may be illustrated by allowing the point F to represent the feed solution and drawing a line FE<sub>1</sub>, of slope  $-(A/B_n)_1$ , to cut the equilibrium curve at E<sub>1</sub>. This then gives composition  $y'_1$  of the extract and  $x'_1$  of the raffinate. If a further stage is then carried out by the addition of solvent **S** to the stream  $A x'_1$ , then point E<sub>2</sub> is found on the equilibrium curve by drawing GE<sub>2</sub> of slope  $-(A/B_n)_2$ . Point E<sub>2</sub> then gives the compositions  $x'_2$  and  $y'_2$  of the final extract and raffinate. This system may be used for any number of stages, with any assumed variation in the proportion of solvent **S** to raffinate from stage to stage.

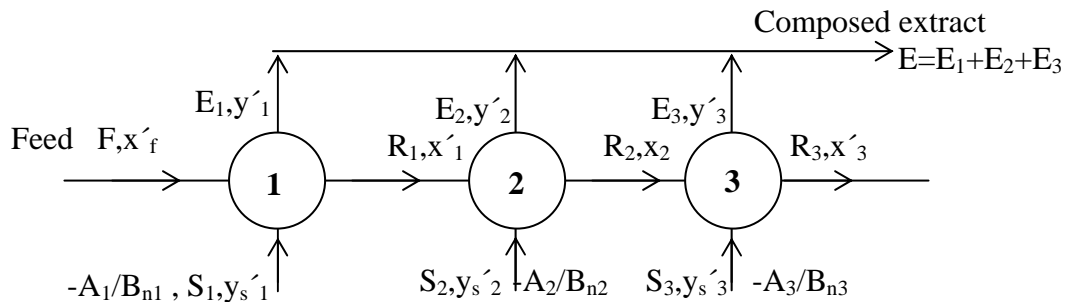


Figure 5.11

If the equilibrium data is a **straight line** ( $y'$  vs.  $x'$ ), to find the number of stages required for separation as :-

1-The equilibrium data is given as  $y' = m x'$ .

2-Make material balance to find the operating line is similar to the condition of equilibrium curve.

C-balance on stage 1

$$A x'_f + B_n y'_s = B_n y'_1 + A x'_1 \dots \dots \dots (23)$$

For pure solvent  $y'_s = 0$

$$y'_{E1} = m x'_{R1} \dots \dots \dots (24a)$$

$$\text{or } y'_1 = m x'_1 \dots \dots \dots (24b)$$

where  $m$  is the slope of equilibrium line; sub equation 24b in equation 23:-

$$A x'_f = B_n m x'_1 + A x'_1 \dots \dots \dots \text{for pure solvent}(y'_s = 0) \\ = x'_1 (m B_n + A)$$

$$x'_1 = \frac{A x'_f}{A + m B_n} \dots \dots \dots (25)$$

For second stage equation 25 become:-

$$x'_2 = \left( \frac{A}{A + m B_n} \right) x'_1 \dots \dots \dots (26)$$

For nth stage equation 26 become:-

$$x'_n = \left( \frac{A}{A + m B_n} \right)^n x'_f \dots \dots \dots (27)$$

$$\text{Or } x'_n = \left( \frac{A}{A + m B_n} \right) x'_{n-1} \dots \dots \dots (28)$$

The number of ideal stages :-

$$n = \frac{\text{Log} \left( \frac{x'_n}{x'_f} \right)}{\text{Log} \left( \frac{A}{A + m B_n} \right)} \dots \dots \dots (29)$$

Equation 29 is used when equilibrium data is straight line and not a curve.

#### 5.5.4. Countercurrent contact with partially miscible solvents.

If a series of mixing and separating vessels is arranged so that the flow is countercurrent, then the conditions of flow may be represented as shown in Figure 5.12, where each circle corresponds to a mixer and a separator. The initial solution  $F$  is fed to the first unit and leaves as raffinate  $R_1$ . This stream passes through the units and leaves from the  $n$ th unit as stream  $R_n$ . The fresh solvent  $S$  enters the  $n$ th unit and passes in the reverse direction through the units, leaving as extract  $E_1$ .

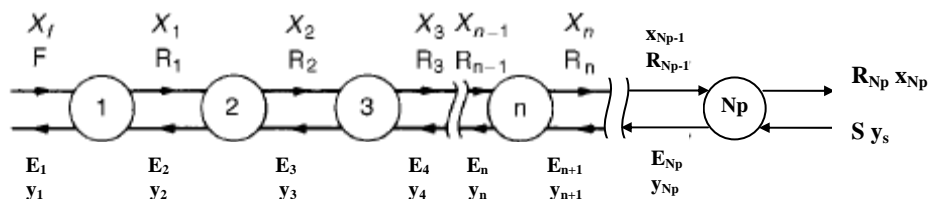


Figure 5.12 Multi-stage counter-current operation.

For a given degree of separation, this type of operation requires fewer stages for a given amount of solvent or less solvent for a fixed number of stages, than the cross-current operation.

**A material balance for the entire plant:-**

$$F + S = E_1 + R_{Np} = M \dots \dots \dots (30)$$

**C-balance:-**

$$F x_f + S y_s = E_1 y_1 + R_{Np} x_{Np} = M x_m \dots \dots \dots (31)$$

From equations 30 and 31, get:-

$$x_m = \frac{F x_f + S y_s}{M} = \frac{F x_f + S y_s}{F + S} \dots \dots \dots (32)$$

Rearrangement equation 30 provides:-

$$R_{Np} - S = F - E_1 = \Delta_R \dots \dots \dots (33)$$

**A material balance for any stage n through  $N_p$  :-**

$$R_{n-1} + S = E_n + R_{Np} \dots \dots \dots (34)$$

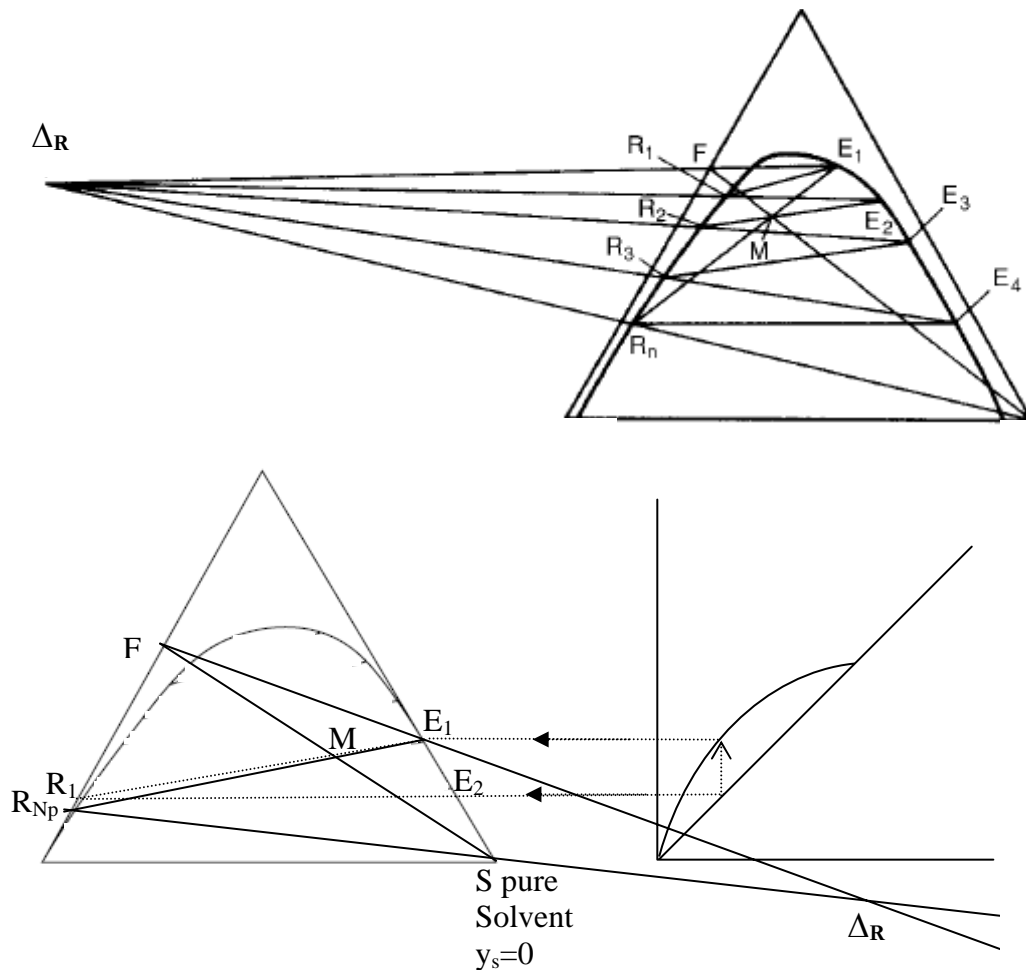
$$R_{n-1} x_{n-1} + S y_s = E_n y_n + R_{Np} x_{Np} \dots \dots \dots (35)$$

$$R_{Np} - S = R_{n-1} - E_n = \Delta_R \dots \dots \dots (36)$$

Where  $\Delta_R$ , difference point and is equal to the net flow outward at the least stage  $N_p$ , for nth stage the difference in quantity between the raffinate leaving a stage  $R_n$ , and the extract entering from next stage  $E_{n+1}$ , is constant. Similarly, it can be shown that the difference between the amounts of each component in the raffinate and the extract streams is constant. This means that, with the notation of a triangular diagram, lines joining any two points representing  $R_n$  and  $E_{n+1}$  pass through a common pole.

Equation 30 shows that  $F$ ,  $S$ ,  $M$  lie on a straight line and also the points  $E_1$ ,  $R_{Np}$ ,  $M$ . Thus the point of intersection of line  $FS$  and  $E_1R_{Np}$  is the point  $M$ . Equation 33 shows that  $\Delta_R$  is the meeting point of line  $F E_1$ ,  $R_{Np}S$ .

In general the difference in flow rates at a location between any two adjacent stages is constant,  $\Delta_R$ , line  $E_n R_{n-1}$  extended must therefore pass through  $\Delta_R$  (lie on a straight line).



The point **F** at  $x_f$  and **S** at  $y_s$  are located, point **M** is located on **FS** line at a mass fraction  $x_m$ . **R<sub>Np</sub>** lies on the solubility curve (binodal curve) at mass fraction  $x_{Np}$ . The line **R<sub>Np</sub>M** is extended to meet the solubility curve at **E<sub>1</sub>**. The line joining **F** and **E<sub>1</sub>** and the line joining **R<sub>Np</sub>** and **S** meet at the  $\Delta_R$  point as is evident from equation 33. since **R<sub>1</sub>** and **E<sub>1</sub>** leave the stage in equilibrium, these must be the ends of a tie line. Point **E<sub>2</sub>** is fixed. From the tie line data  $x_1$  corresponding to  $y_1$  is found and the point **R<sub>1</sub>** located on the solubility curve at mass fraction  $x_1$ . Equation 33 shows that  $\Delta_R$ , **R<sub>1</sub>** and **E<sub>2</sub>** lie on a straight line. The line  $\Delta_R$  is extended to meet the solubility curve at **E<sub>2</sub>**. **R<sub>2</sub>E<sub>2</sub>** is a tie line. Corresponding to  $y_2$ ,  $x_2$  is found from the tie line data, and the point **R<sub>2</sub>** located on the solubility curve. the line  $\Delta_R$  is now extended to meet the solubility curve at **E<sub>3</sub>**. The procedure is repeated till the tie line passes through point **R<sub>Np</sub>** or cross it, then determine the number of ideal stages required for separation.

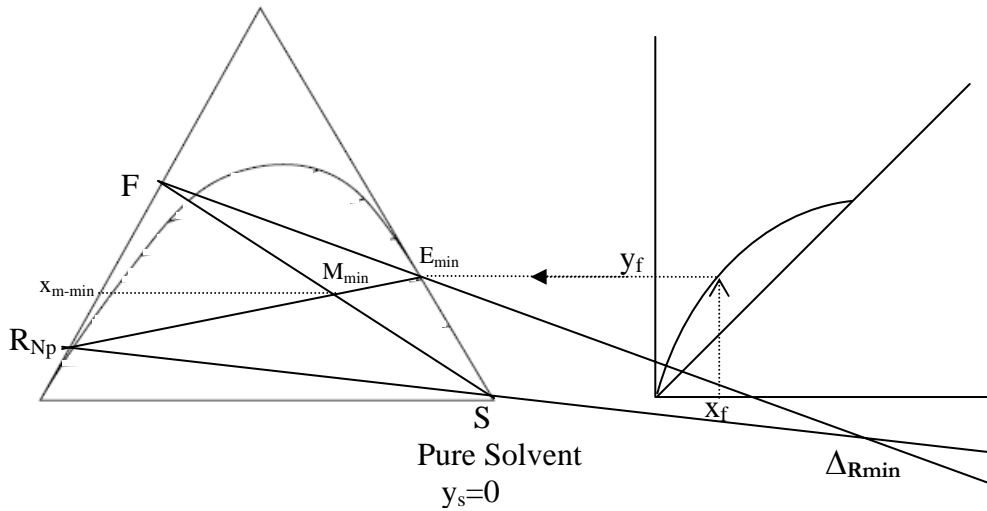
### **Minimum Solvent Required.**

Minimum solvent required for the specified product can be determined by locating the  $\Delta_{Rmin}$  point for minimum solvent. After locating **S** at  $y_s$ , **F** at  $x_f$ , **R<sub>Np</sub>** at  $x_{Np}$ , the equilibrium composition  $y_f$  at the feed condition, locate on the solubility curve (**E<sub>min</sub>**) from the distribution curve.

The line **E<sub>min</sub>F** is extended to line **S R<sub>Np</sub>**, to give intersections with line **S R<sub>Np</sub>** in  $\Delta_{Rmin}$  as shown in figure below. The intersection farthest from **S** (if  $\Delta$  point is on



the left side of the diagram) or nearest S (if  $\Delta$  point is on the right side of the diagram) .



The minimum( $x_{m-min}$ ) is located by joining ( $R_{Np}$ ,  $E_{min}$ ) and ( $F$ ,  $S$ ), the point of intersection is  $M_{min}$ , then  $S_{min}$  is calculated:-

$$F + S_{min} = M_{min} \dots \dots \dots (37)$$

$$F x_f + S_{min} y_s = M_{min} x_{m-min} \dots \dots \dots (38)$$

$$x_{m-min} = \frac{F x_f + S y_s}{F + S_{min}} \text{ or } \frac{F x_f}{F + S_{min}} \text{ (for pure solvent)} \dots \dots \dots (39)$$

### 5.5.5. Countercurrent contact with immiscible solvents.

When the liquid A and B are insoluble over the range of solute concentrations encountered, the stage computation is made more simply on  $x'$ ,  $y'$  coordinates. For this case, the solvent content of all extracts and the A content of all raffinate are constant.

$$\begin{aligned} \text{For the 1st stage:} & \quad A x'_f + B_n y'_2 = A x'_1 + B_n y'_1 \\ \text{For the nth stage:} & \quad A x'_{n-1} + B_n y'_{n+1} = A x'_n + B_n y'_n \\ \text{For the whole unit:} & \quad A x'_f + B_n y'_s = A x'_{Np} + B_n y'_1 \dots \dots \dots (40). \end{aligned}$$

$$\frac{A}{B_n} = \frac{(y'_1 - y'_s)}{(x'_f - x'_{Np})} \dots \dots \dots (41)$$

Equation 40 is the operating line of slope ( $A/B_n$ ), which passes through the points ( $x'_f$ ,  $y'_1$ ) and ( $x'_{Np}$ ,  $y'_s$ ). In Figure 5.13, the equilibrium relation,  $y'_n$  against  $y'_n$ , and the operating line are drawn in, and the number of stages required to pass from  $x'_f$  to  $x'_{Np}$  is found by drawing in steps between the operating line and the equilibrium

curve. In this example of Figure 5.13 , four stages are required, and  $(x'_{Np}, y'_s)$  corresponds to  $(x'_4, y'_5)$ . It may be noted that the operating line connects the compositions of the raffinate stream leaving and the fresh solvent stream entering a unit,  $x'_{Np}$  and  $y'_s$ , respectively.

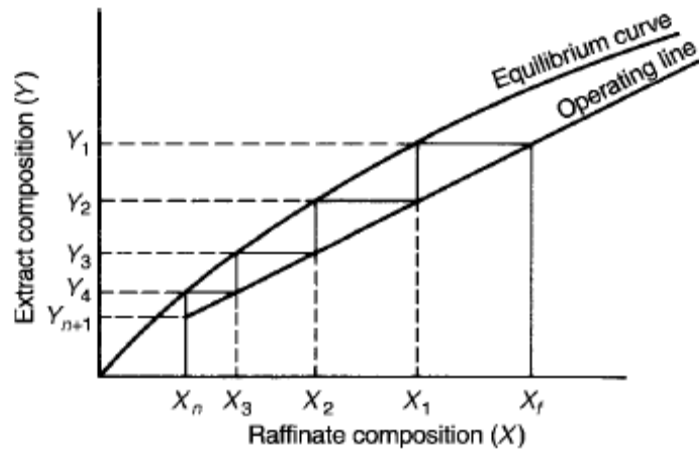


Figure5.13

Similarly equation 41 can be written for stage through n as :-

$$\frac{A}{B_n} = \frac{(y'_1 - y'_{n+1})}{(x'_f - x'_n)} \dots\dots\dots(42)$$