Decomposition in BioSignal Processing

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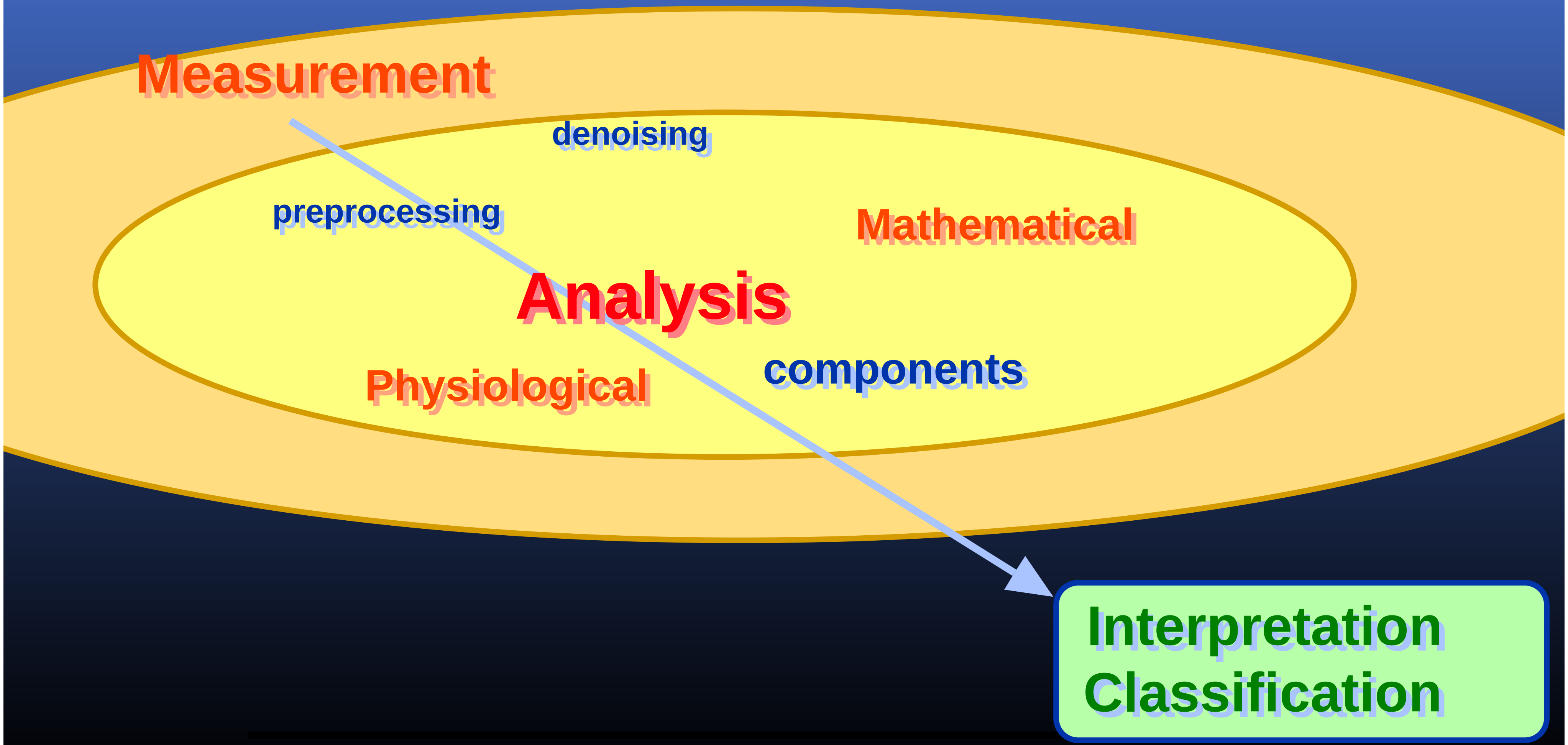
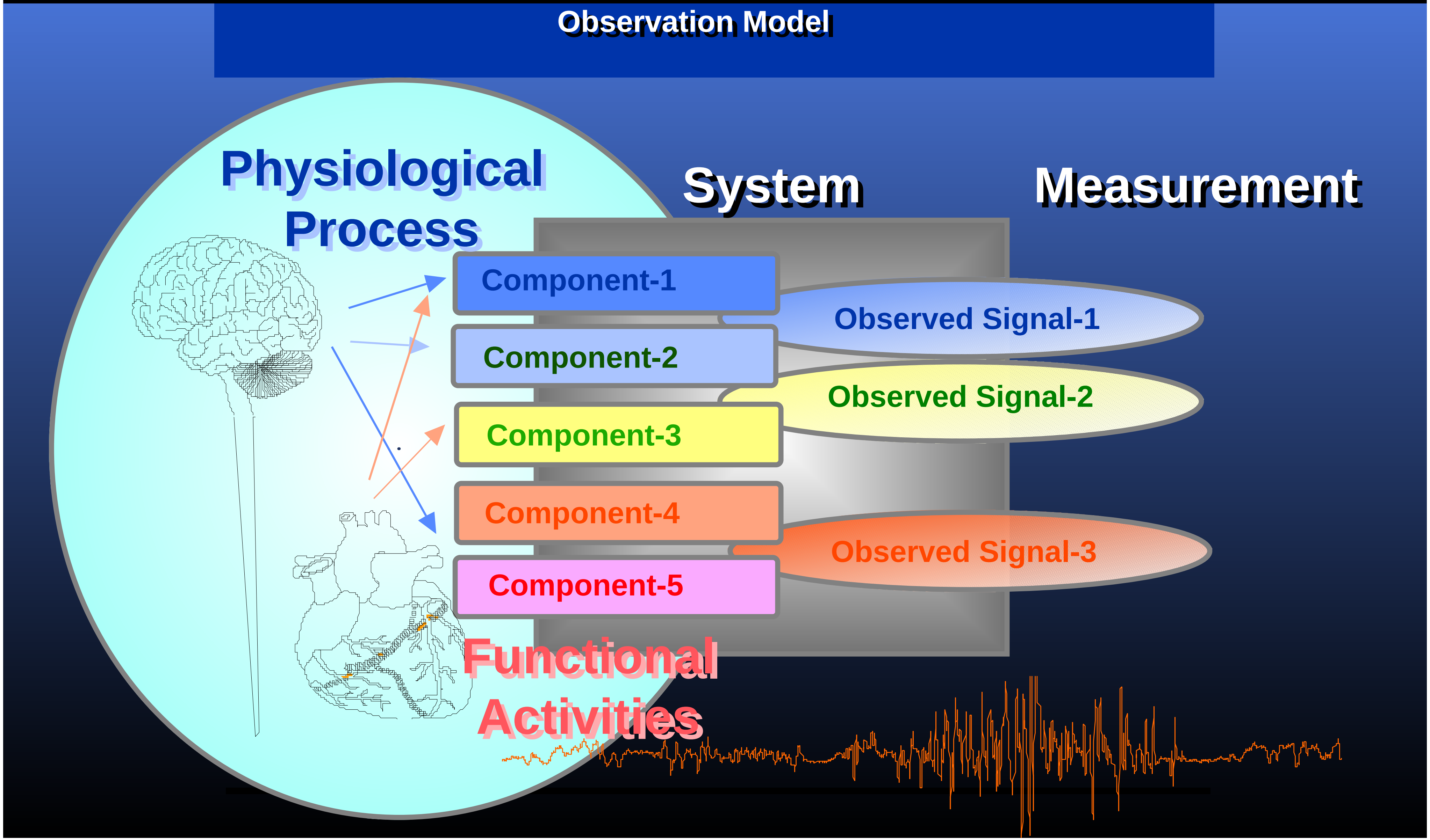
Abstract: For decomposing a biosignal into its constituents, there have been two major approaches in this field: physiological and mathematical approaches. Decomposition of action potentials from an interference signal is an example of physiological decomposition. On the other hand, the Fourier transform decomposes a measured signal into complex exponential signal set, mathematically. Recently, some attractive approaches have been proposed for handling nonlinearity and nonstationarity of biosignals. Among them, the Multiple Signal Classification method, the Independent Component Analysis, and Matching Pursuit method in Wavelet analysis are described in this literature.

Location of Decomposition

in BioSignal Processing



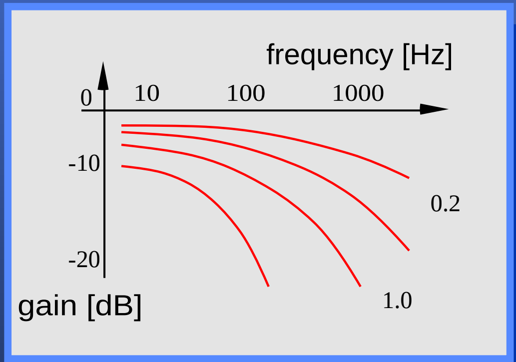
Observation



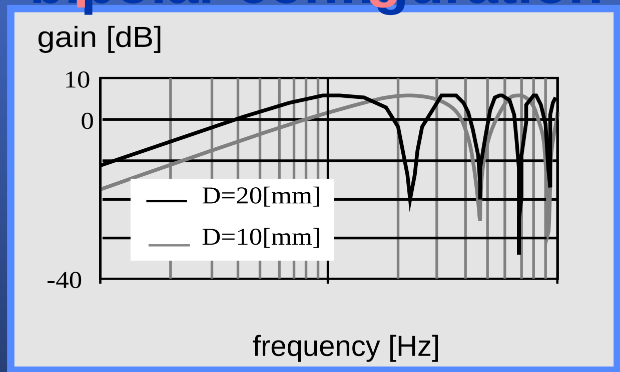
Measurement

System Function

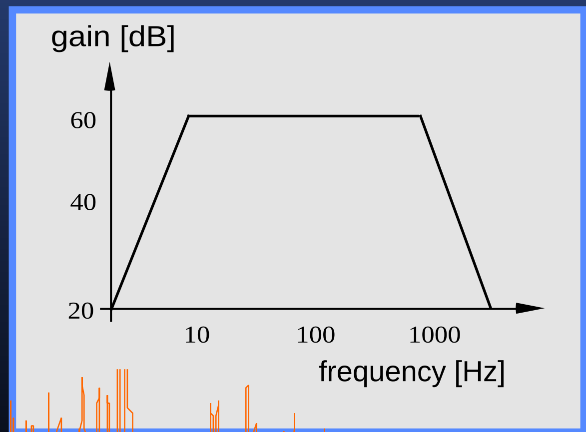
tissue filter

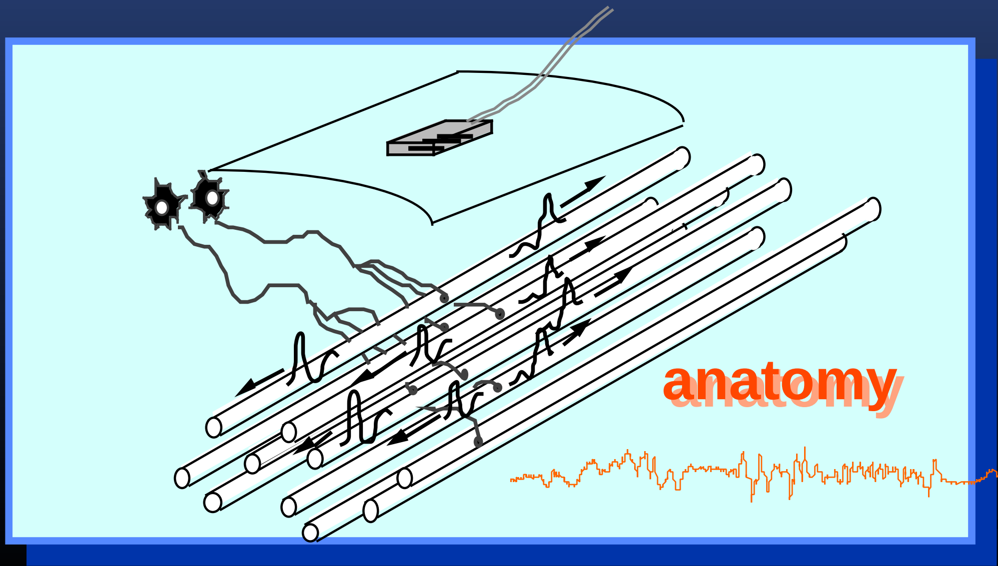


bipolar configuration filter



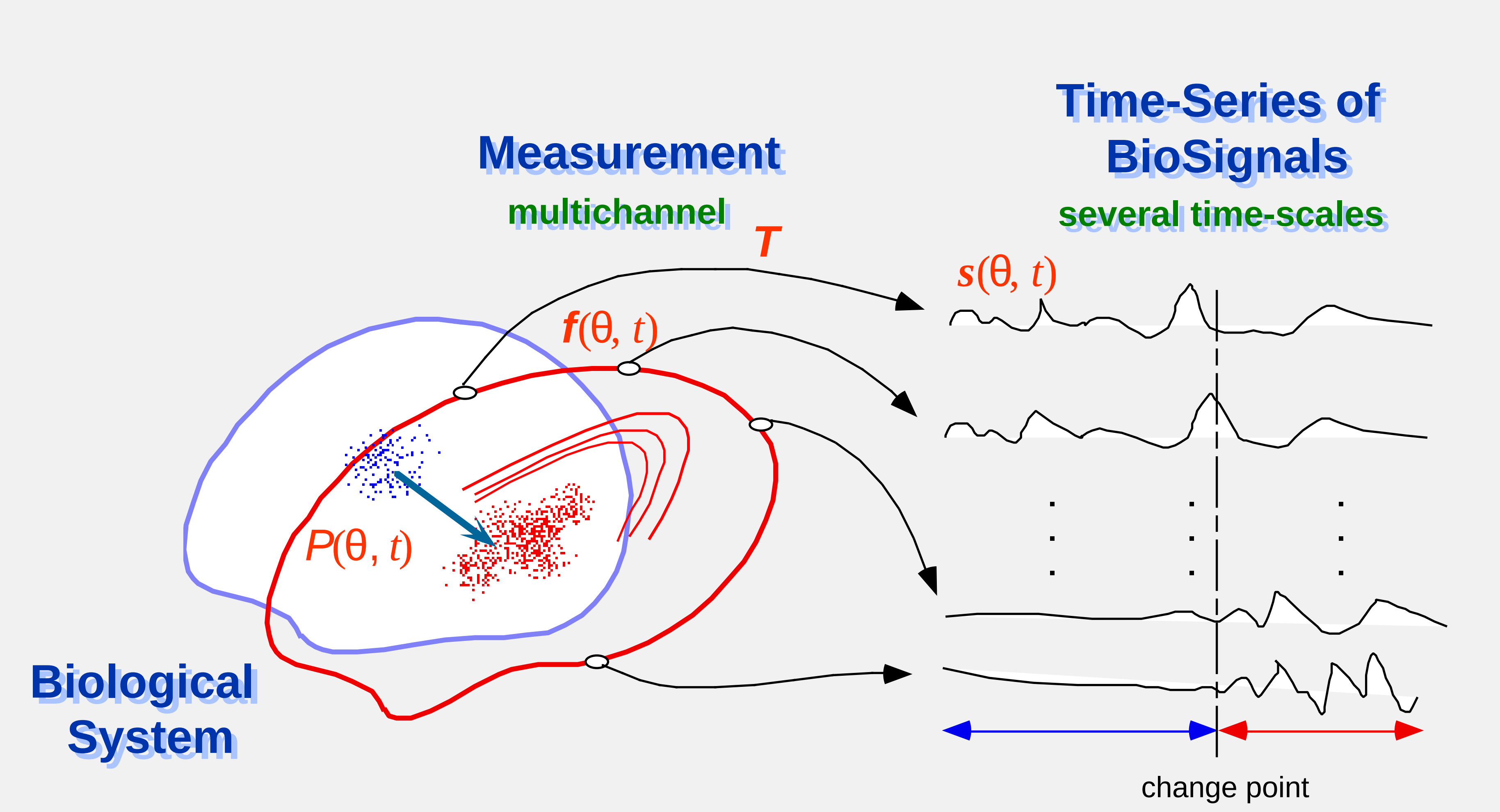
amplifier





Biosignals

Process



Two Major Approaches

depending on the Aim

- Mathematical Approach

orthogonal

φ

complex exponential

$e^{j\omega t}$

- Physiological Approach

perception

functional activities

mechanics

physiology

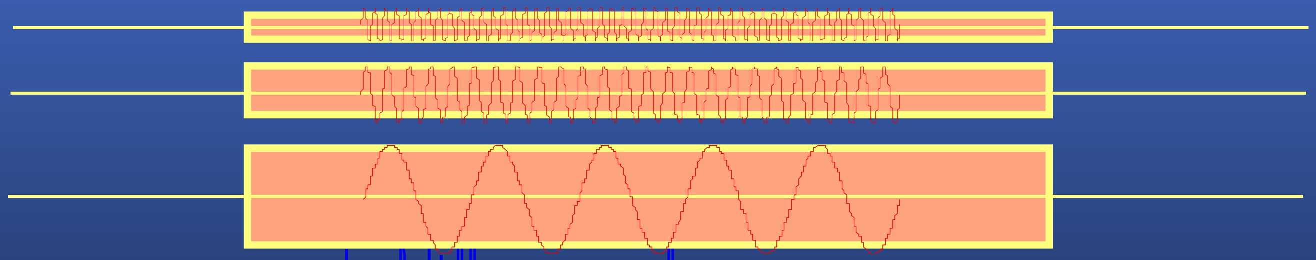
nervous system

anatomy

Components

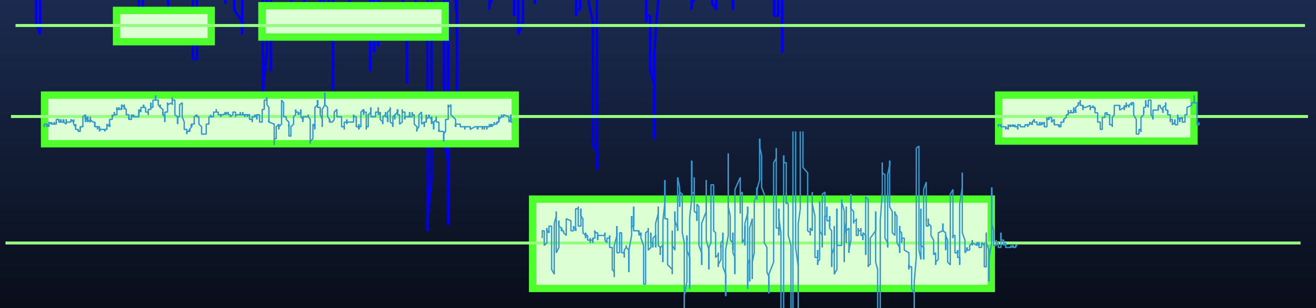
Time Domain

- Decomposition for Analysis



mathematical components

- Decomposition for Classification

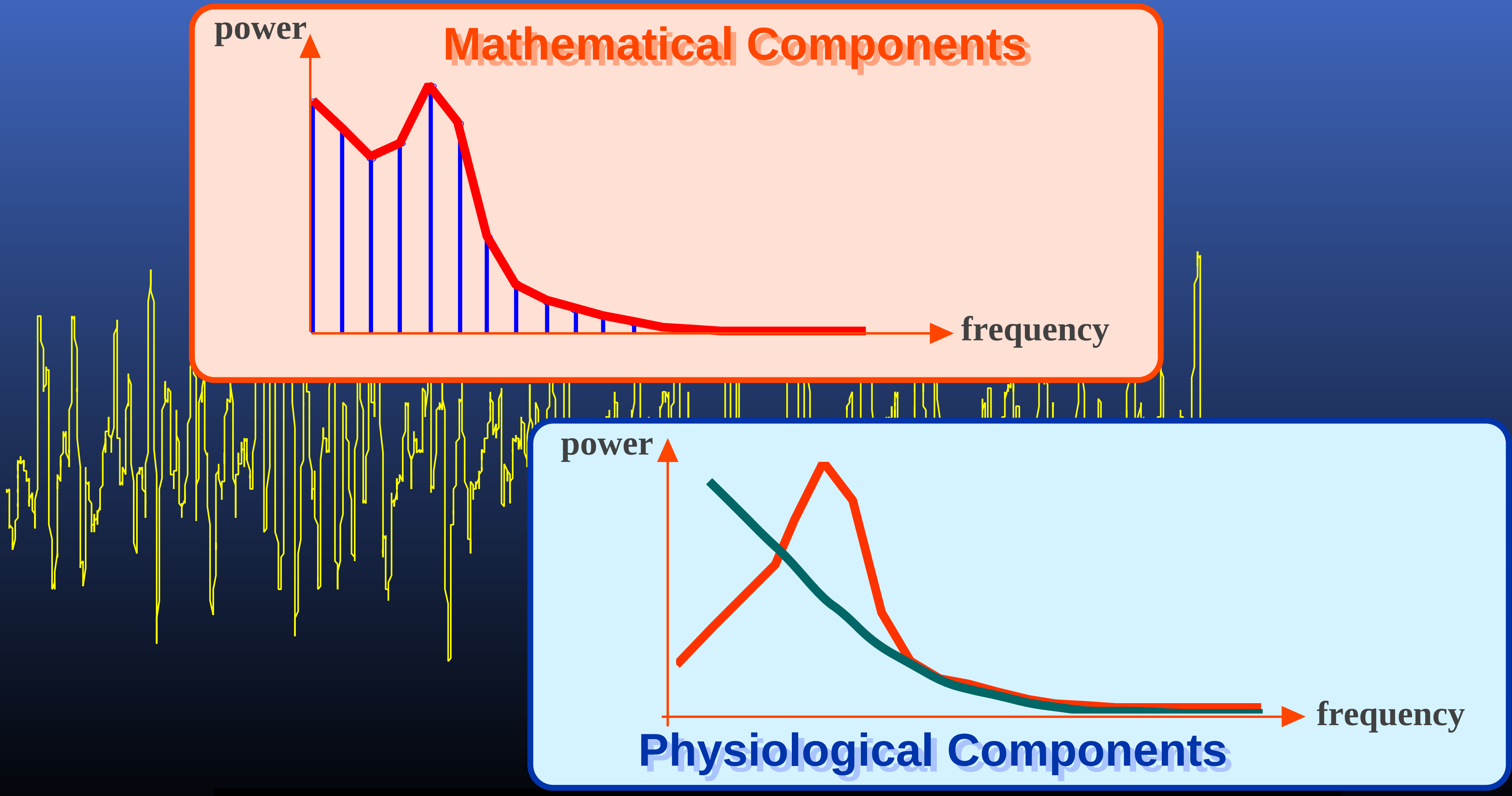


physiological components

BPES99 in Kobe Univ., Oct. 7, 1999

Components

Frequency Domain



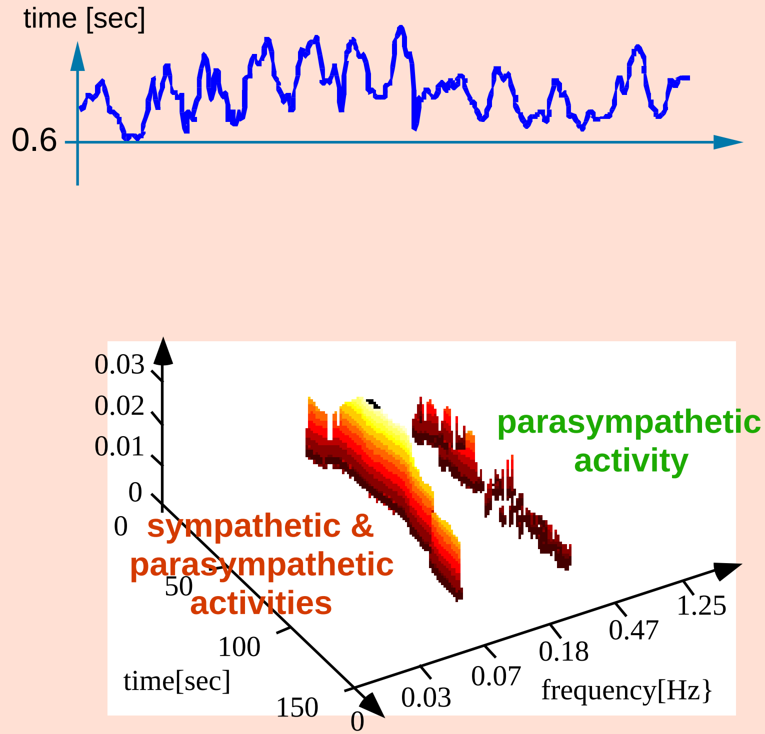
Mathematical Approach

Fourier Transform

- Periodical Components

- signal and noise
- fluctuation

$$y(t) = \sum_{k=\infty}^{\infty} Y_k e^{jk\omega_0 t}$$



example of heart rate variability

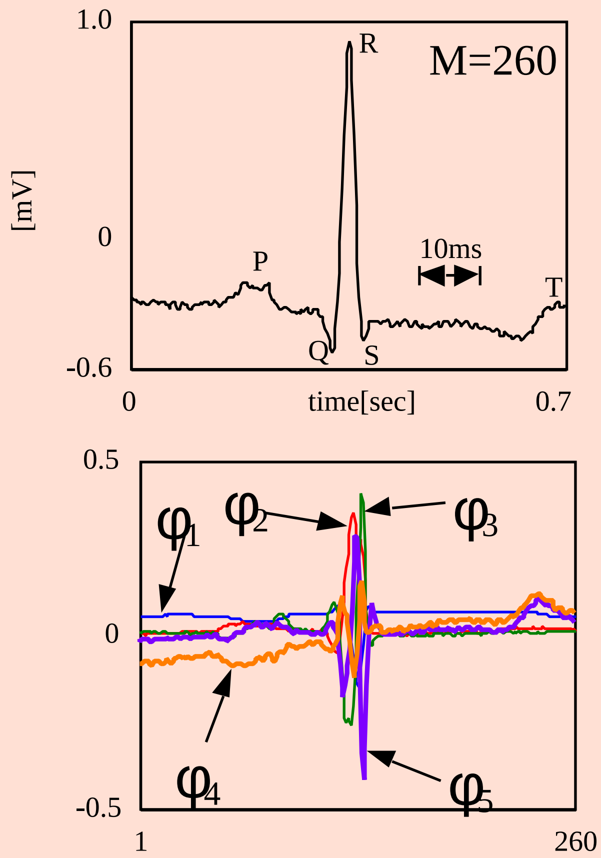
Mathematical Approach

PCA

- Orthogonal Components

- signal and noise
- data compression

$$y_{(K)} = \sum_{k=1}^{K} (\boldsymbol{y}^T \Phi_k)\Phi_k$$

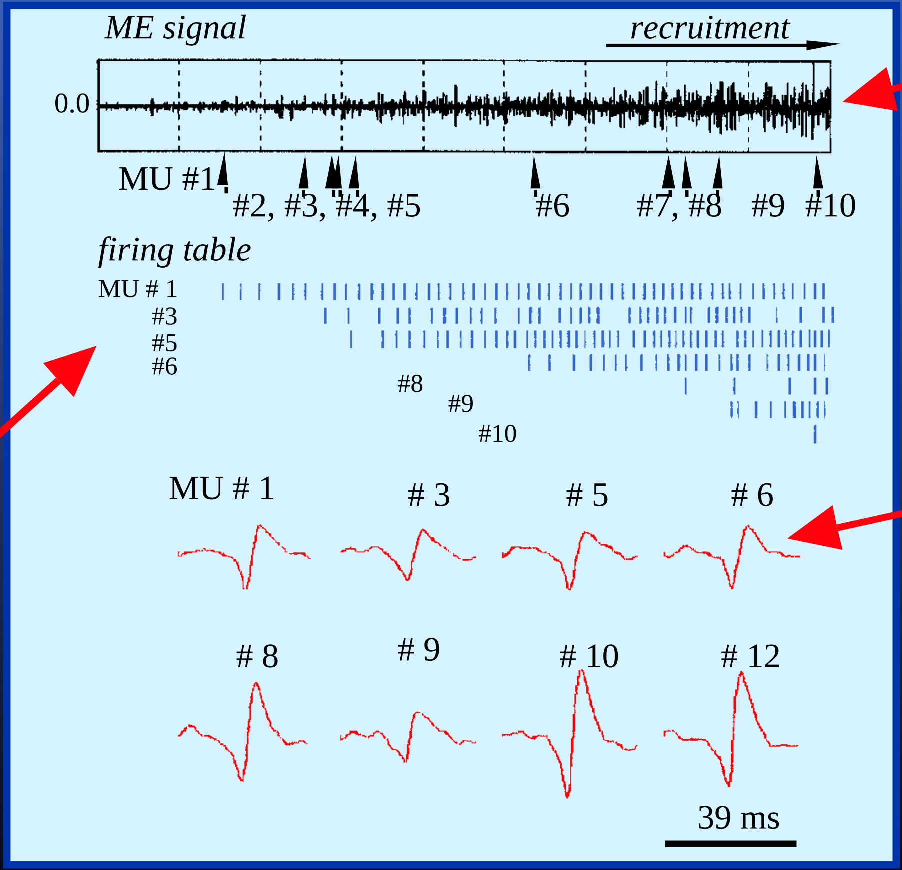


example of electrocardiogram

Physiological Approach

Motor Unit Decomposition

- Physiological Components



Measured Signal

$$x(t) = \sum_{k=1}^{K} \alpha_k u_k(t)$$

motor unit action potentials

- fast-twitch, slow-twitch MUs

firing table

R. LeFever and C. J. De Luca: A procedure for decomposing the myoelectric signal into its constituent action potentials part I: Technique, theory, and implementation, IEEE Trans. BME, Vol. BME-29, 3, 149/157 (1982).

MU decomposition. An interference myoelectric signal is decomposed into its firing table of motor unit action potentials (MUAPs) by statistical analysis.

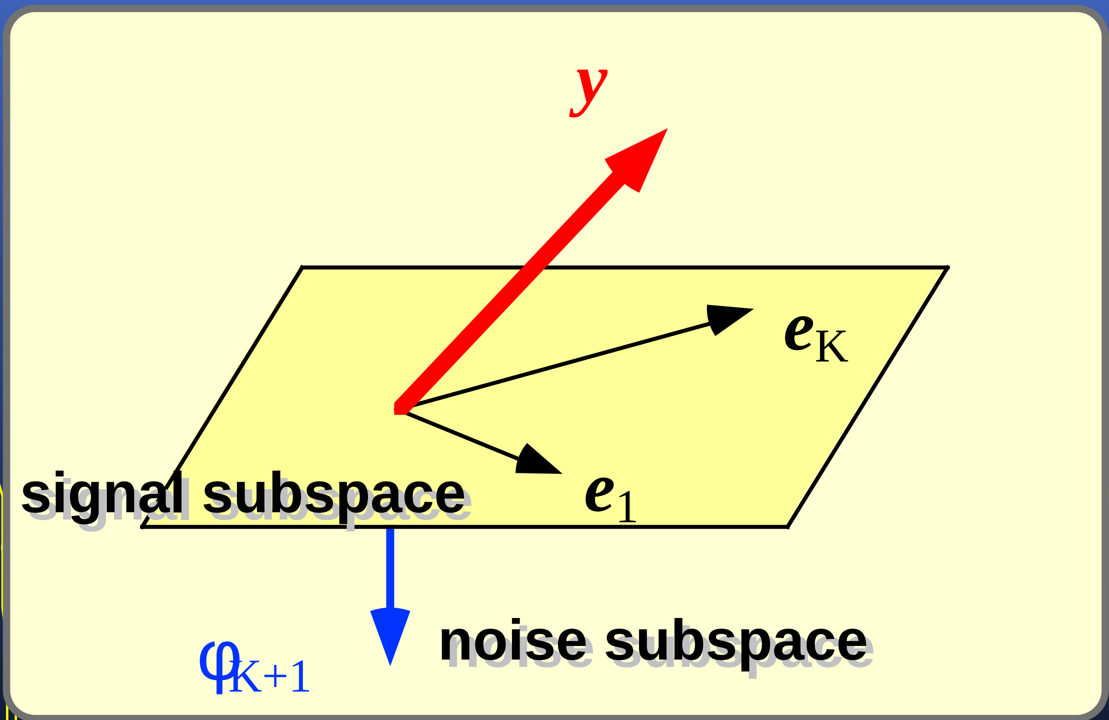
New Approaches

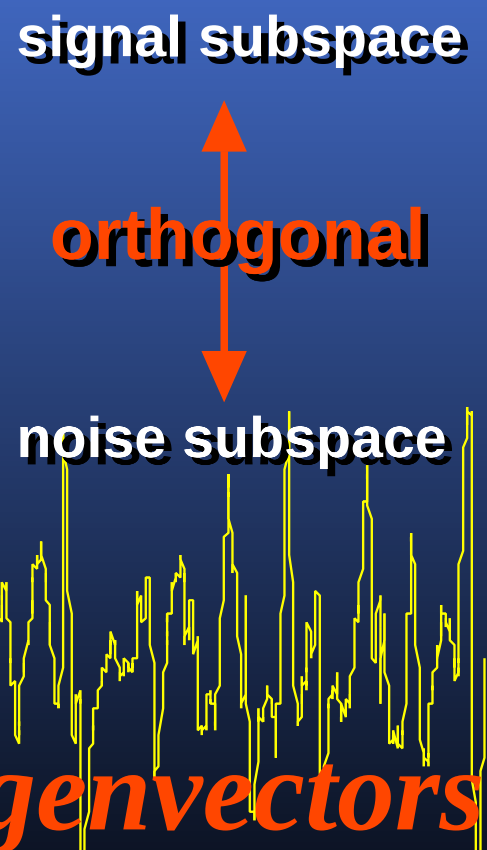
... from around 1990

- Multiple Signal Classification
- Independent Component Analysis
- Matching Pursuit in Wavelet Analysis

MUSIC

Multiple Signal Classification





Overview of MUSIC method. An eigenvector in noise subspace is orthogonal to the signal subspace.

References for MUSIC

from 1979

- V. F. Pisarenko: On the estimation of spectra by means of non linear functions of the covariance matrix, *Geophys. J. R. Astron. Soc.*, 28, 511(1972)
- R. Schmidt: Multiple emitter location and signal parameter estimation, *Proc. RADC Spectrum Estimation Workshop*, pp. 243-258 (1979).
- M. Akay: Detection and estimation methods for biomedical signals, *Academic Press* (1996).

Subspaces

for MUSIC

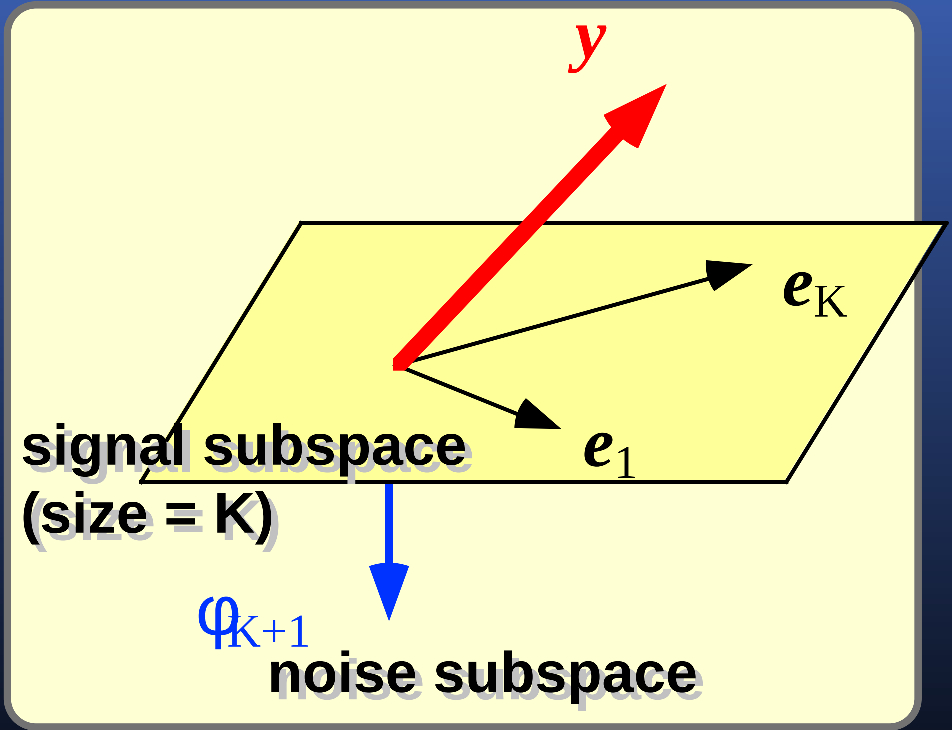
Observation Model

$$y(n) = \sum_{k=1}^{K} a_k e^{j\omega_k n} + \varepsilon(n)$$

- orthogonal

$$e_k = (1\ e^{j\omega_k}\ e^{j2\omega_k} \ldots e^{j(N-1)\omega_k})$$

$$(\boldsymbol{e}_k^*)^T \varphi_{K+1} = 0, \quad k = 1, 2, \ldots, K$$

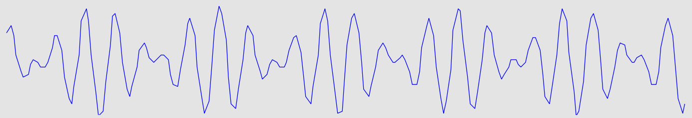


Power Spectrum

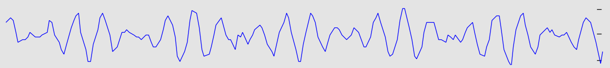
$$P_{MU}(\omega) = \frac{1}{\sum_{i=K+1}^{L} \left| \{\boldsymbol{e}^*(\omega)\}^T \varphi_i \right|^2}$$

Example of MUSIC

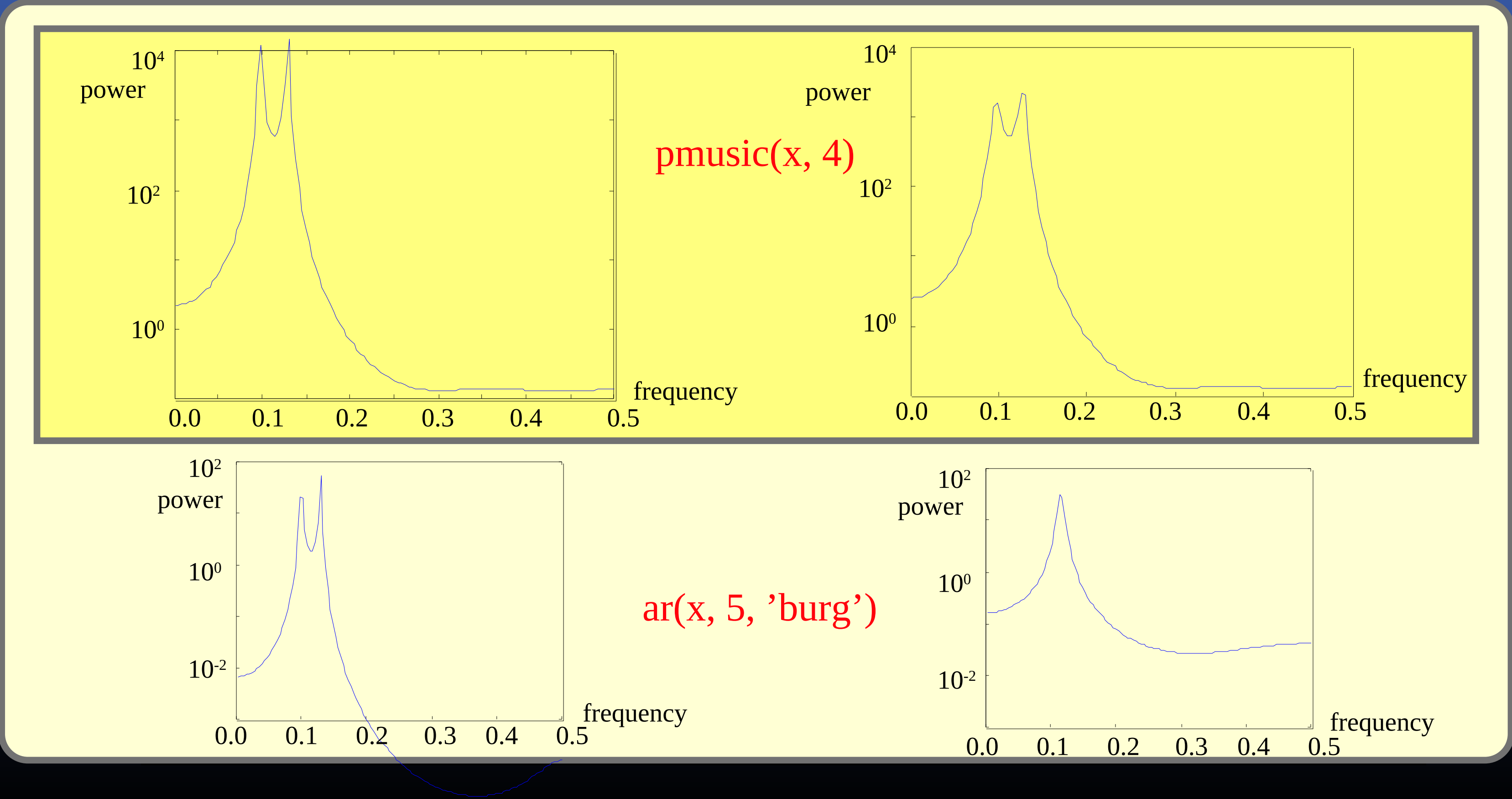
Estimation of Frequency Components from Noisy Signal



cos(0.257*pi*n)+sin(0.2*pi*n)+0.01*randn(size(n))



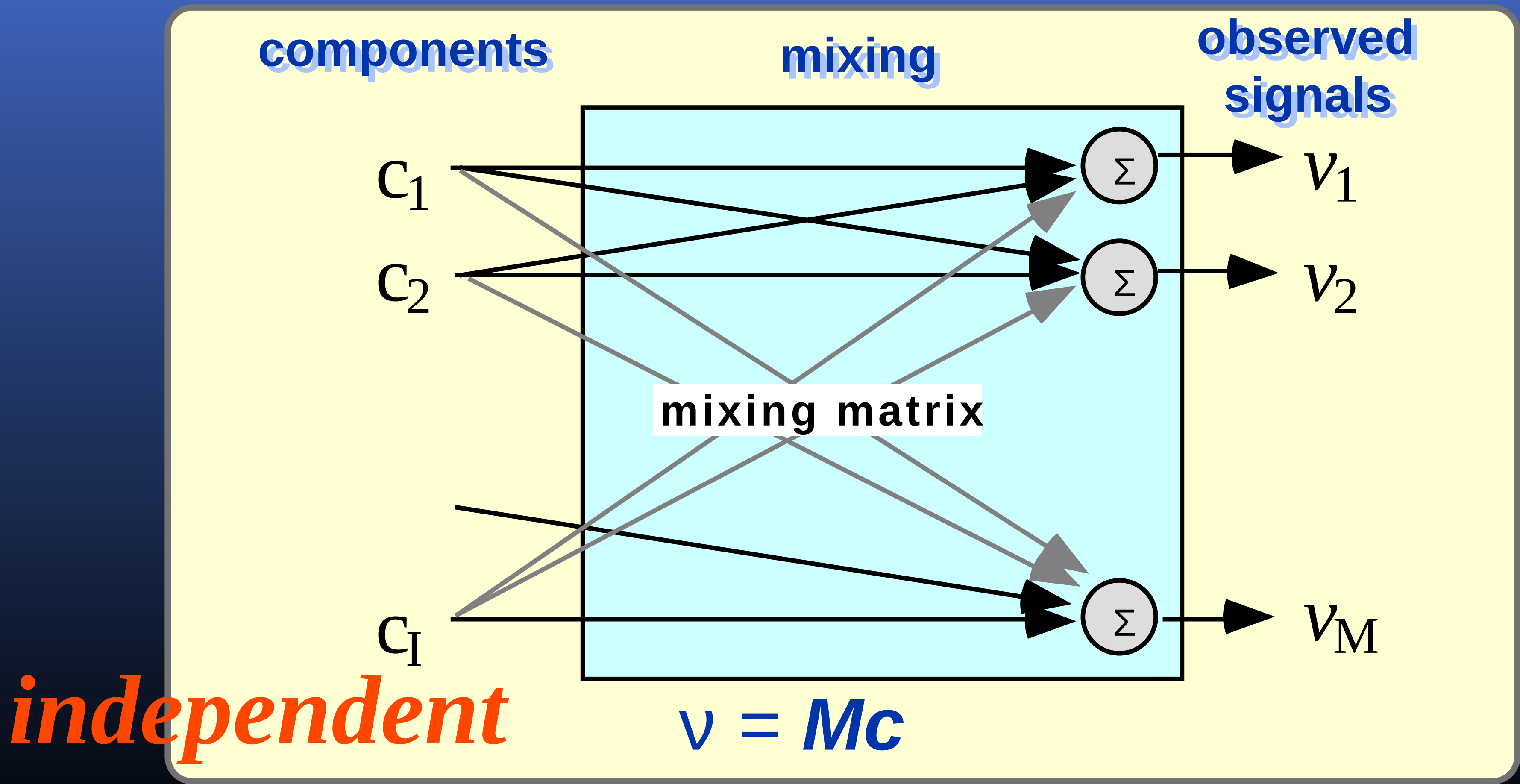
cos(0.257*pi*n)+sin(0.2*pi*n)+0.2*randn(size(n))



ICA

Independent Component Analysis for Blind Separation

Observation Model



Overview of the ICA Multichannel observed signal is presumed to be composed of statistically independent components.

Reference for ICA

from 1995

- A. J. Bell and T. J. Sejnowski: An information maximization approach to blind separation and blind deconvolution, *Neural Comput*, Vol. 7, 6, 1129/1159 (1995).
- M. J. McKeown, S. Makeig, G. G. Brown, T. P. Jung, S. S. Kindermann, A. J. Bell, and T. J. Sejnowski: Analysis of fMRI data by blind separation into independent spatial components, *Hum Brain Mapp*, vol. 6, 3, 160/188 (1998).

Statistically Independent

for ICA

Statistical Definition

- independent

$$p(x_1, x_2, ..., x_M) = p(x_1)p(x_2) \quad p(x_M)$$

- uncorrelated

$$E[x_k\, x_l] = E[x_k]E[x_l]$$

$$E[(x_k - \mu_k)(x_l - \mu_l)] = 0$$

- orthogonal

$$E[x_k\, x_l] = 0$$

Estimation Algorithm

$$\Delta W = \lambda\{\boldsymbol{I} + f(\boldsymbol{c})\boldsymbol{c}^T\}\boldsymbol{W}$$

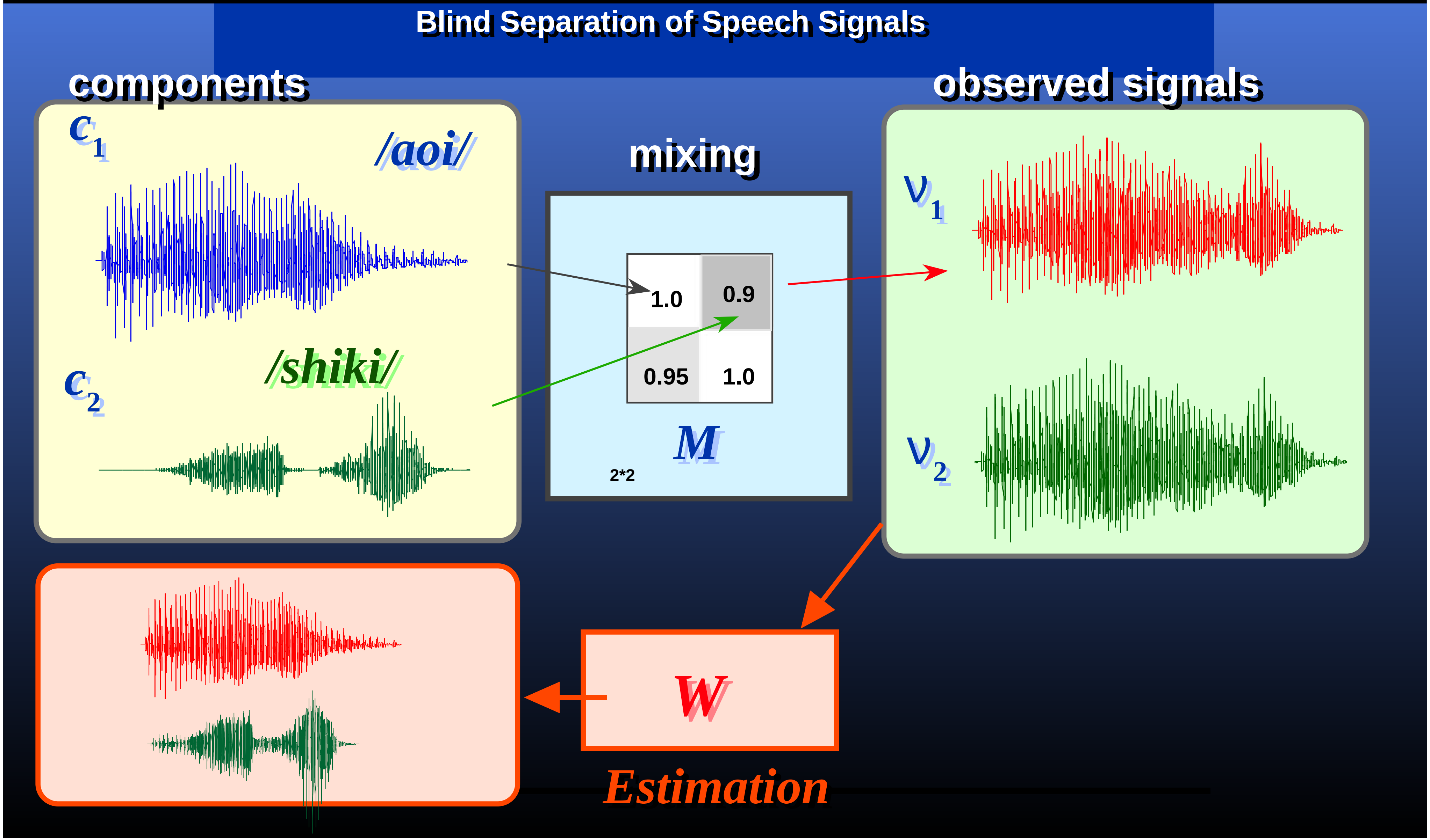
$$f(\boldsymbol{c}) = 1 - \frac{2}{1 + exp(-\boldsymbol{c})}$$

$$\hat{\boldsymbol{c}} = \boldsymbol{W}\nu_s$$

$$\nu_s = \frac{2\nu}{\sqrt{(\nu\nu^T)}}$$

http://www.cnl.salk.edu/~scott/ica-download-form.html

Example of ICA



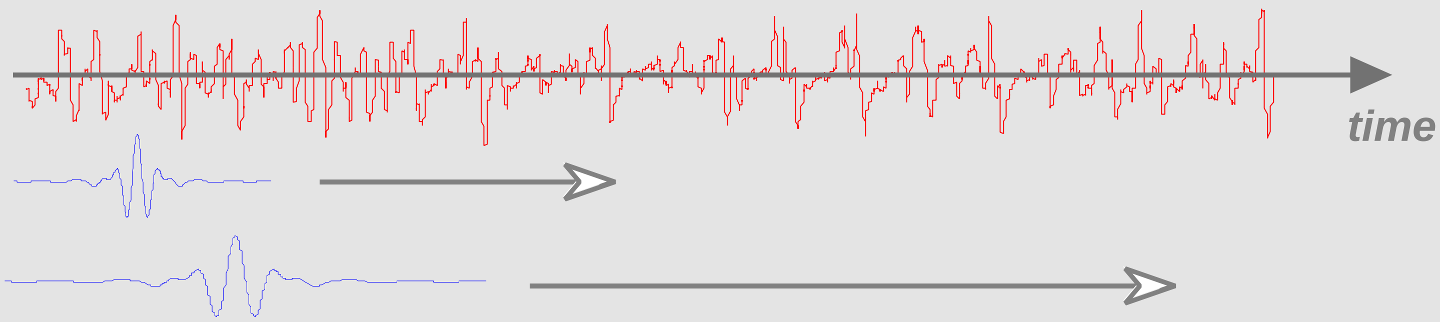
Continuous Wavelet Transform

Time-Scale Analysis

Definition

$$W(a,b) = \int_{-\infty}^{\infty} \psi^{*}_{a,b}(t)\,y(t)\,dt$$

- procedure



Wavelet Function

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\,\psi\left(\frac{t-b}{a}\right)$$

Discrete Wavelet Transform

Wavelet Decomposition

Observation Model

$$y(t) \approx \sum_j \sum_k d_k^{(j)} \psi(2^j t - k)$$

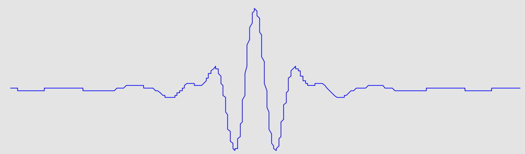
- decomposition

$$y(t) = A_J(t) + D_J(t) + D_{J-1}(t) + \dots + D_1(t)$$

Two-scale Relation

- wavelet function

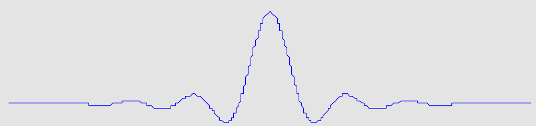
$$\psi(t) = \sum_{k=1}^{\infty} q_k \phi(2t - k)$$



Meyer Wavelet

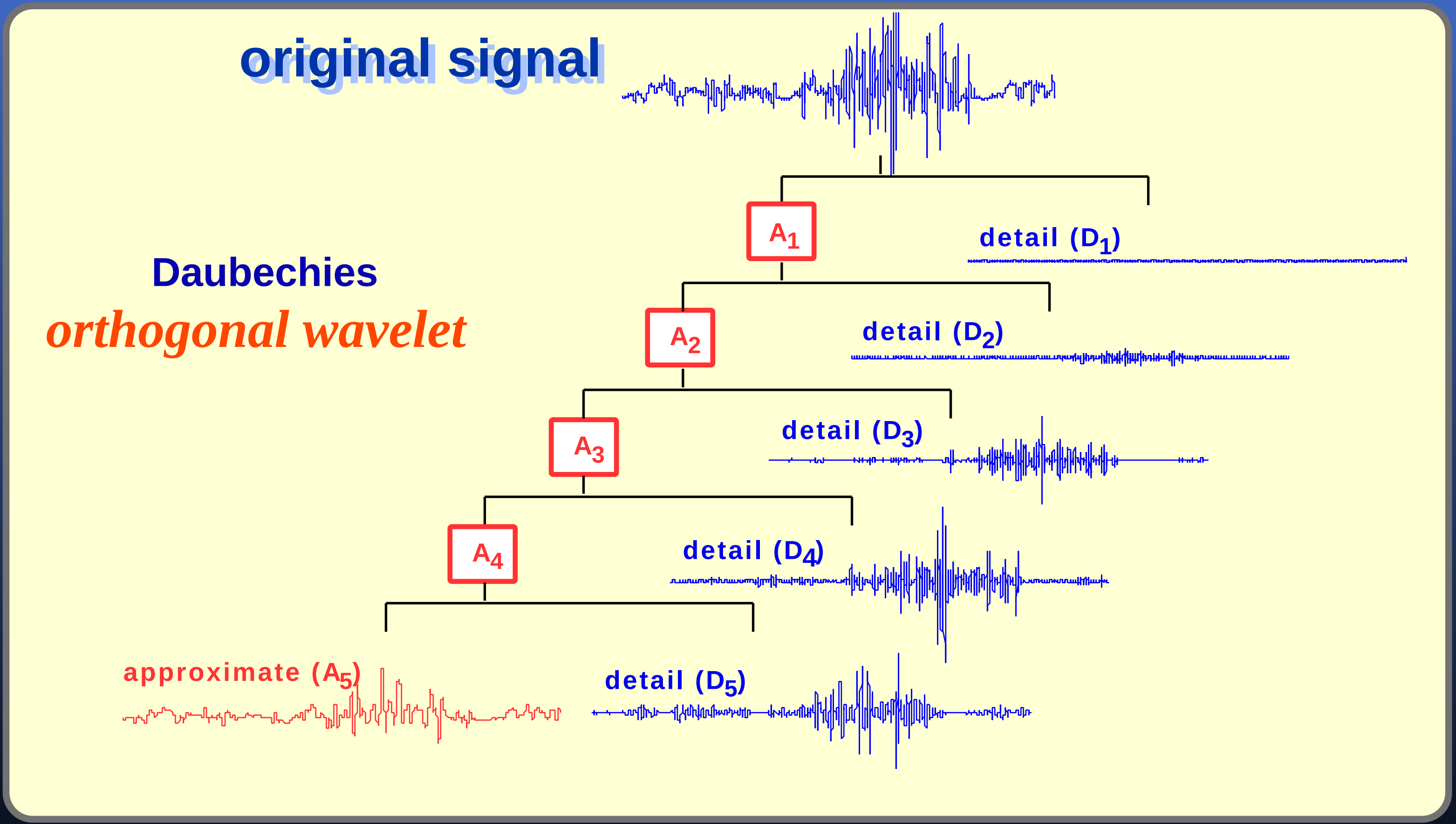
- scaling function

$$\phi(t) = \sum_{k=1}^{\infty} r_k \phi(2t - k)$$



Example of Wavelet Decomposition

Approximates and Details



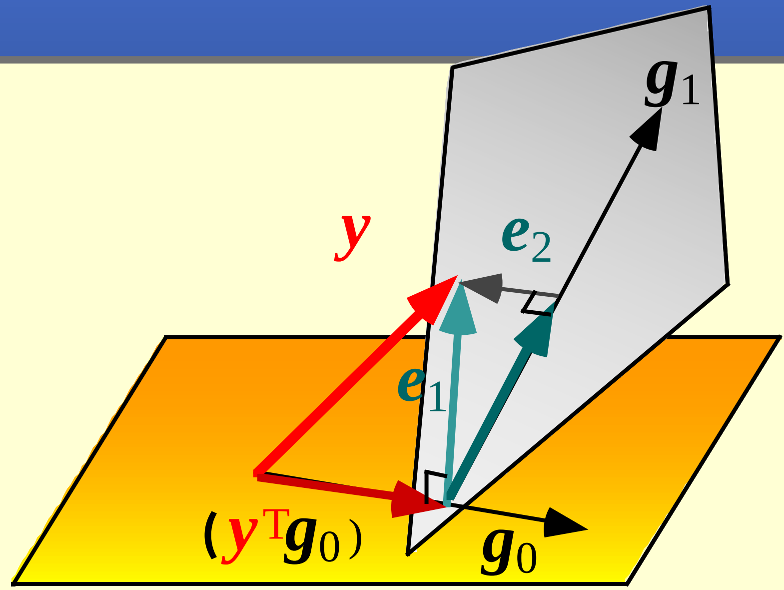
Example of multiresolution analysis in the discrete Wavelet transform. Using the Daubechies-5, a surface myoelectric signal is decomposed into its approximate (A5) and some details (D1, ..., D5).

Matching Pursuit

Estimation of Several Types of Frequency Components

Observation Model

$$\boldsymbol{y} = \sum_{i=1}^{K-1} \left[\left(\boldsymbol{R}^{i} \boldsymbol{y} \right)^{\mathrm{T}} \boldsymbol{g}_{\gamma_i} \right] \boldsymbol{g}_{\gamma_i} + \boldsymbol{R}^{K} \boldsymbol{y}$$



$$\boldsymbol{y} = (\boldsymbol{y}^{\mathrm{T}}\boldsymbol{g}_0)\boldsymbol{g}_0 + \boldsymbol{e}_1$$
$$\boldsymbol{e}_1 = (\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{g}_1)\boldsymbol{g}_1 + \boldsymbol{e}_2$$

- time-frequency atom

$$\boldsymbol{g}_{\gamma_i} = \frac{1}{\sqrt{s_i}} g\left(\frac{t-u_i}{s_i}\right) e^{j\zeta_i t}$$

$$\left\| \boldsymbol{g}_{\gamma_i} \right\| = 1$$

orthogonal

I am just studying this approach. Don't ask me the detail!

Reference for MP

from 1993

- S. G. Mallat and S. Zhang: Matching pursuits with time-frequency dictionaries, *IEEE Trans SP*, Vol. 41, 12, 3397/3415 (1993).
- M. Akay: Detection and estimation methods for biomedical signals, *Academic Press* (1996).

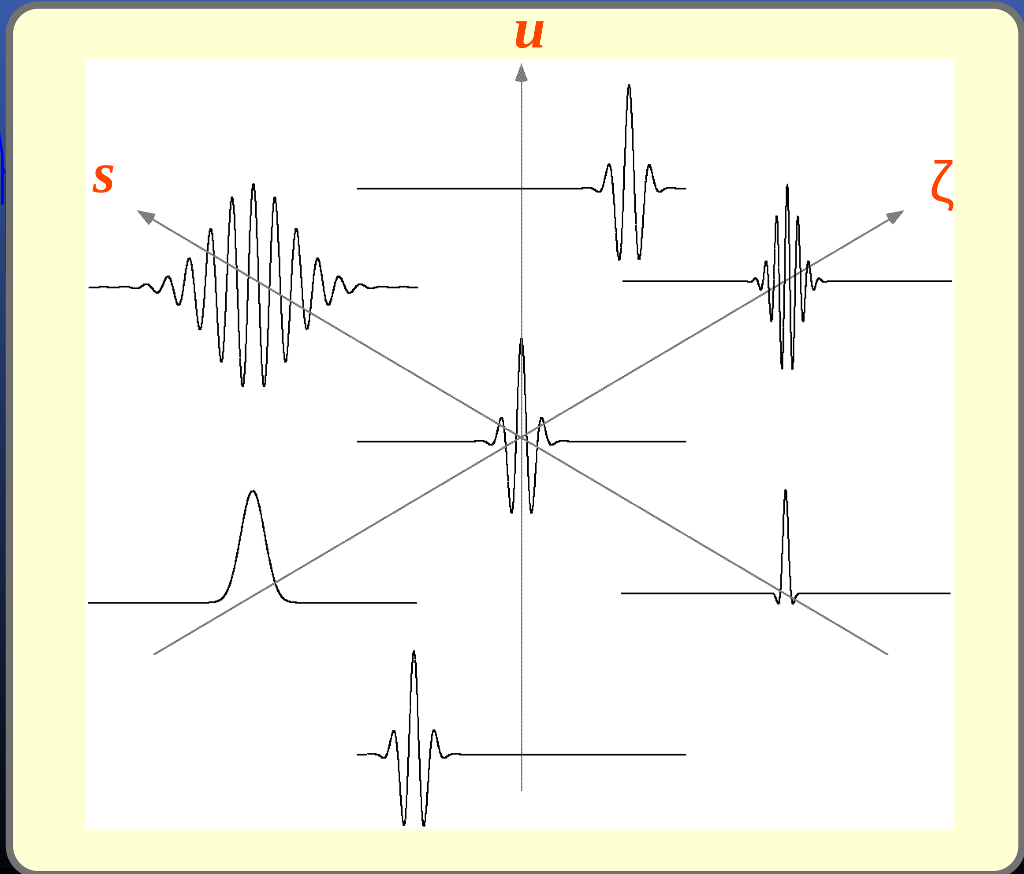
Wavelet Dictionary

for Matching Pursuit

$$\boldsymbol{g}_{\gamma_i} = \frac{1}{\sqrt{s_i}} g\left(\frac{t-u_i}{s_i}\right) e^{j\zeta_i t}$$

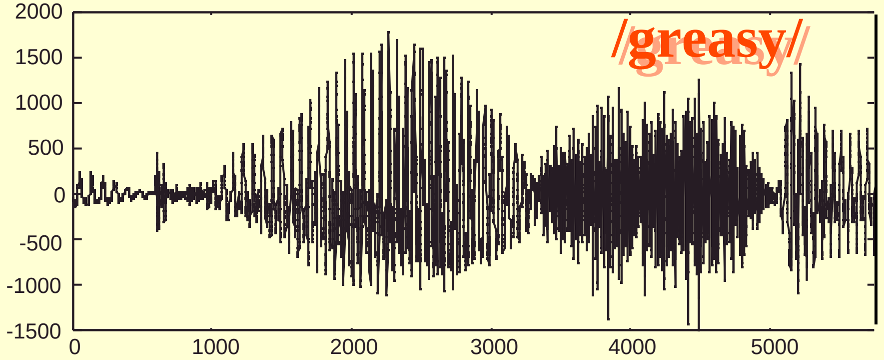
$$\gamma_i = (s_i, u_i, \zeta_i)$$

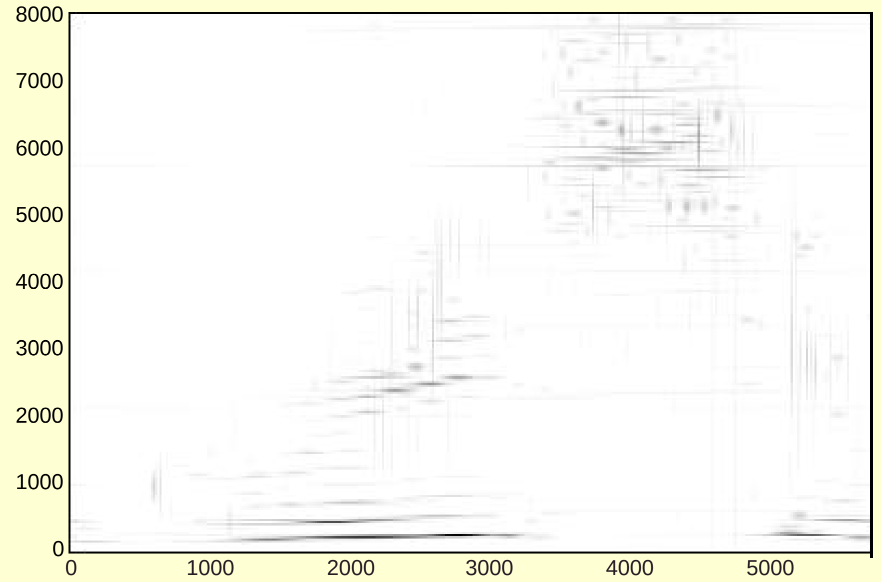
Gabor time-frequency atoms



Example of Matching Pursuit

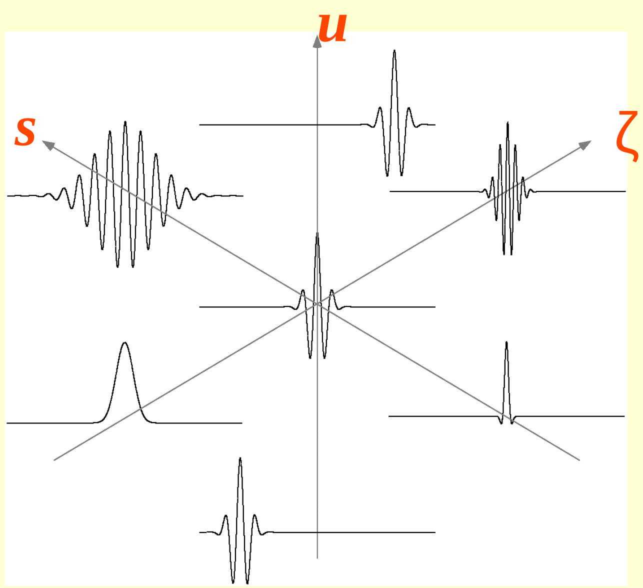
Analysis of Speech Signals





$$\boldsymbol{y} = \sum_{i=1}^{K-1} \left[\left(\boldsymbol{R}^i \boldsymbol{y} \right)^{\mathrm{T}} \boldsymbol{g}_{\gamma_i} \right] \boldsymbol{g}_{\gamma_i} + \boldsymbol{R}^K \boldsymbol{y}$$

Gabor time-frequency atoms



ftp://cs.nyu.edu/

S. G. Mallat and S. Zhang: Matching pursuits with time-frequency dictionaries, *IEEE Trans SP*, Vol. 41, 12, 3397/3415 (1993).

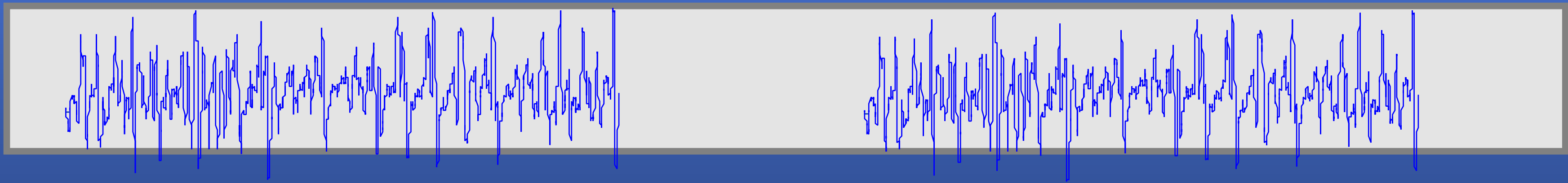
Conclusion

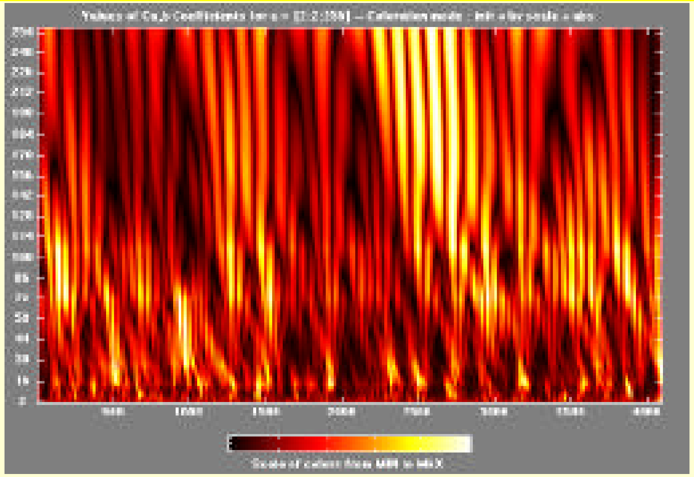
Decomposition Procedure

- Understanding observation model.
- Selecting approaches (physiological or mathematical approaches).
- Estimation of components.
- Interpretation of the time-varying behavior of components.

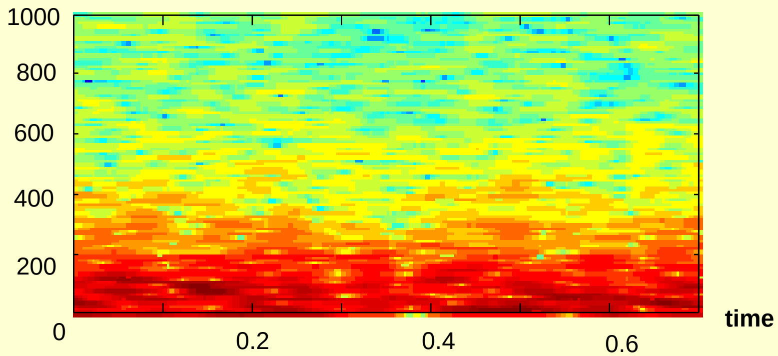
Example

physiological data

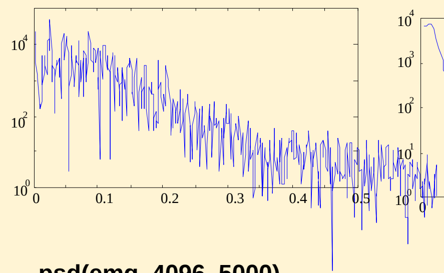




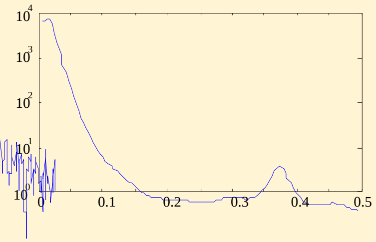
CWT:
db6, 2:2:256, int+by scale+abs



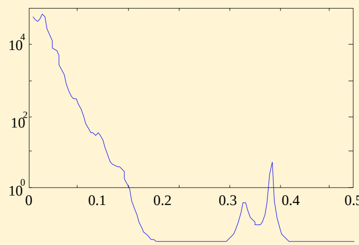
specgram(emg, 512, 5000, kaiser(500,5), 475)



psd(emg, 4096, 5000)



ar(emg,15,'burg')



pmusic(emg,15)